

Calcolo Scientifico

a.a. 2007-2008

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Problema ...

Sistemi triangolari

Sistemi triangolari inferiori

Sistemi triangolari superiori

$$\begin{cases} a_{00} x_0 = b_0 \\ a_{10} x_0 + a_{11} x_1 = b_1 \\ a_{20} x_0 + a_{21} x_1 + a_{22} x_2 = b_2 \end{cases}$$

$$\begin{cases} a_{00} x_0 + a_{01} x_1 + a_{02} x_2 = b_0 \\ a_{11} x_1 + a_{12} x_2 = b_1 \\ a_{22} x_2 = b_2 \end{cases}$$

Forward Substitution

(x_0, x_1, x_2)

Backward Substitution

(x_2, x_1, x_0)

Sistemi triangolari inferiori ...

$$\begin{cases} a_{00} x_0 & = b_0 \\ a_{10} x_0 + a_{11} x_1 & = b_1 \\ a_{20} x_0 + a_{21} x_1 + a_{22} x_2 & = b_2 \\ \vdots & \vdots \\ a_{n0} x_0 + a_{n1} x_1 + a_{n2} x_2 + \dots + a_{nn} x_n & = b_n \end{cases}$$

Algoritmo di Forward Substitution

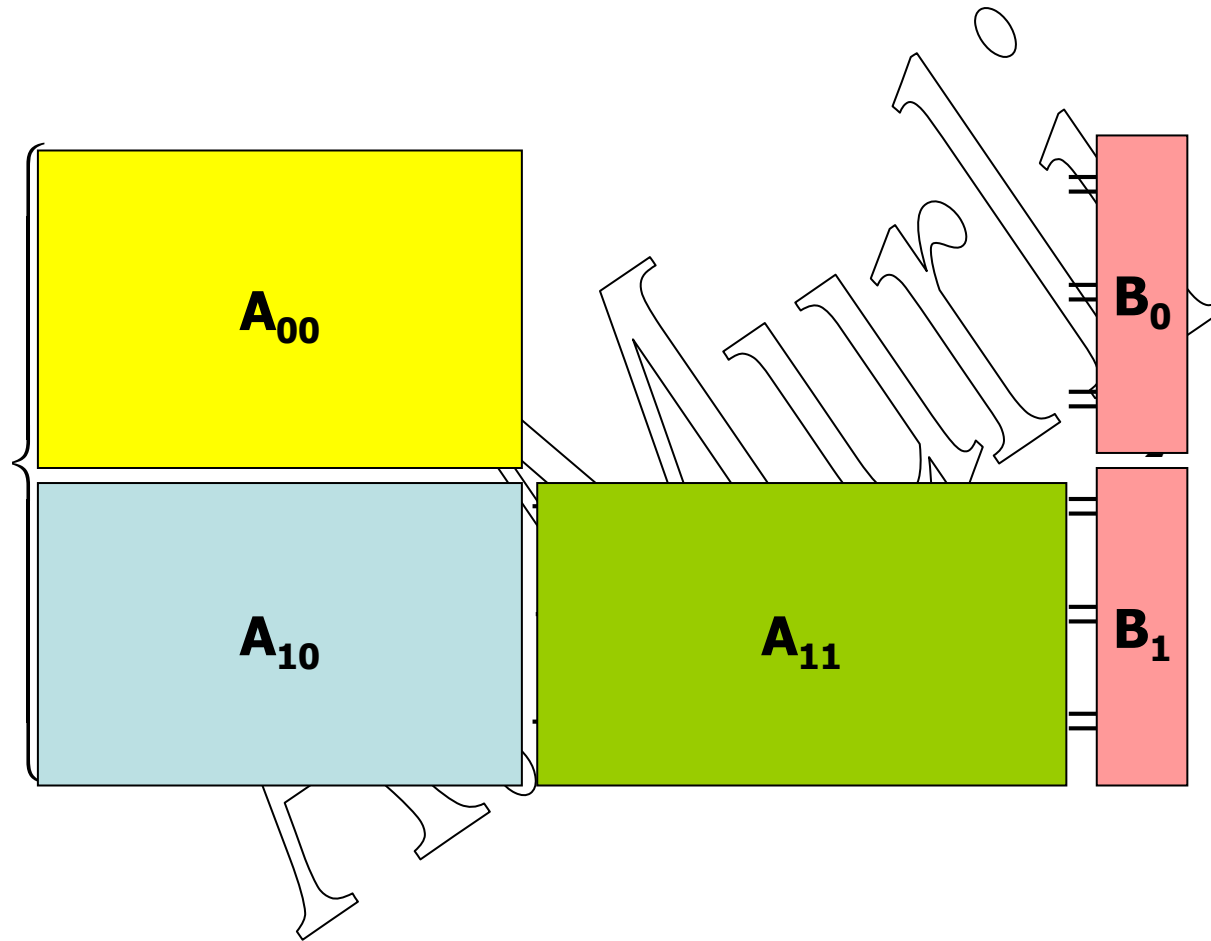
$$\begin{cases} x_0 = b_0 / a_{00} \\ x_i = \left(b_i - \sum_{j=0}^{i-1} a_{ij} x_j \right) / a_{ii} \end{cases}$$

$$\forall i = 1, \dots, n$$

Consideriamo un sistema di dimensione $n=6$ e 2 blocchi 3×3

$$\begin{cases} a_{00} x & = b_0 \\ a_{10} x + a_{11} x & = b_1 \\ a_{20} x + a_{21} x + a_{22} x & = b_2 \\ a_{30} x + a_{31} x + a_{32} x + a_{33} x & = b_3 \\ a_{40} x + a_{41} x + a_{42} x + a_{43} x + a_{44} x & = b_4 \\ a_{50} x + a_{51} x + a_{52} x + a_{53} x + a_{54} x + a_{55} x & = b_5 \end{cases}$$

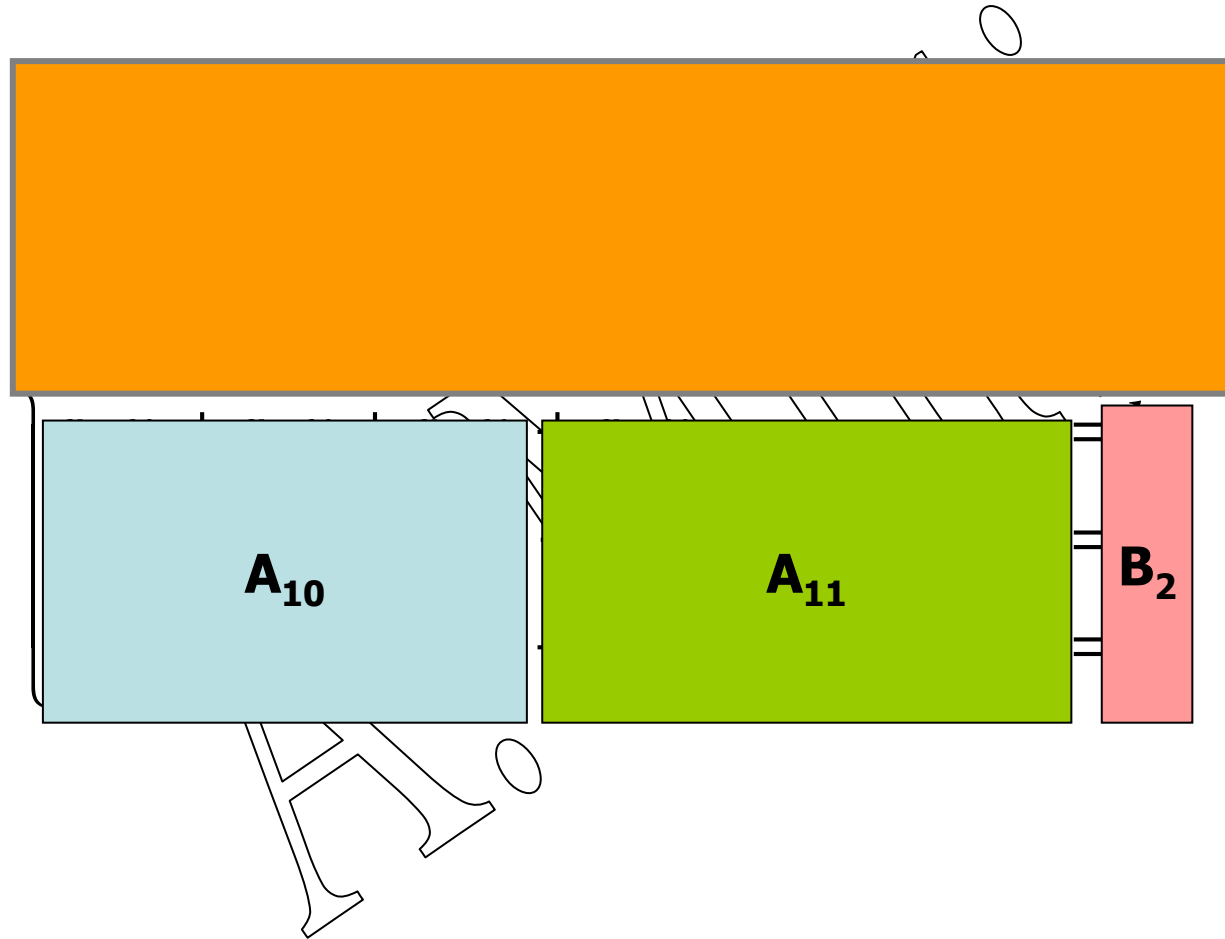
Partizioniamo la matrice a blocchi di dimensione 3 x 3



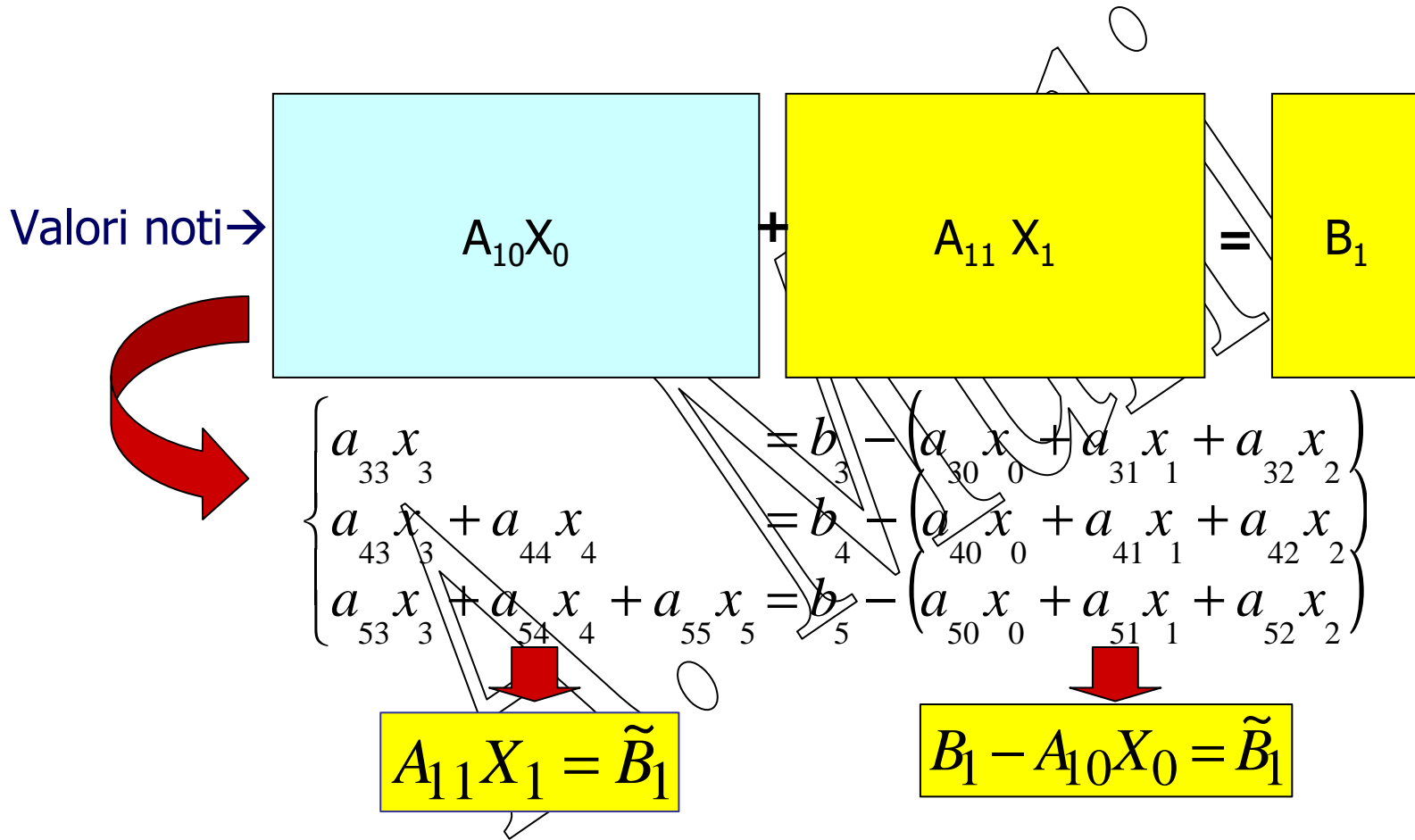
Si risolve il sistema $A_{00}X_0 = B_0$

$$\begin{cases} a_{00} x_0 = b_0 \\ a_{10} x_0 + a_{11} x_1 = b_1 \\ a_{20} x_0 + a_{21} x_1 + a_{22} x_2 = b_2 \\ a_{30} x_0 + a_{31} x_1 + a_{32} x_2 + a_{33} x_3 = b_3 \\ a_{40} x_0 + a_{41} x_1 + a_{42} x_2 + a_{43} x_3 + a_{44} x_4 = b_4 \\ a_{50} x_0 + a_{51} x_1 + a_{52} x_2 + a_{53} x_3 + a_{54} x_4 + a_{55} x_5 = b_5 \end{cases}$$

Si considerano i blocchi rimanenti $A_{10}X_0 + A_{11}X_1 = B_1$



Dopo aver calcolato X_0 Si modifica il termine noto B_1



Consideriamo un sistema di dimensione $n=6$ e 3 blocchi 2×2

$$\begin{array}{l}
 \boxed{
 \begin{array}{l}
 a x_{00} = b_0 \\
 a x_{10} + a x_{11} = b_1
 \end{array}
 } \\
 \boxed{
 \begin{array}{l}
 a x_{20} + a x_{21} + a x_{22} = b_2 \\
 a x_{30} + a x_{31} + a x_{32} + a x_{33} = b_3
 \end{array}
 } \\
 \boxed{
 \begin{array}{l}
 a x_{40} + a x_{41} + a x_{42} + a x_{43} + a x_{44} = b_4 \\
 a x_{50} + a x_{51} + a x_{52} + a x_{53} + a x_{54} + a x_{55} = b_5
 \end{array}
 }
 \end{array}$$

Si risolve il sistema $A_{00} X_0 = B_0$

$$\begin{cases}
 a_{00} x_0 = b_0 \\
 a_{10} x_0 + a_{11} x_1 = b_1 \\
 a_{20} x_0 + a_{21} x_1 + a_{22} x_2 = b_2 \\
 a_{30} x_0 + a_{31} x_1 + a_{32} x_2 + a_{33} x_3 = b_3 \\
 a_{40} x_0 + a_{41} x_1 + a_{42} x_2 + a_{43} x_3 + a_{44} x_4 = b_4 \\
 a_{50} x_0 + a_{51} x_1 + a_{52} x_2 + a_{53} x_3 + a_{54} x_4 + a_{55} x_5 = b_5
 \end{cases}$$

A

Si modifica il termine noto B_1 (b_2, b_3)

$$\begin{cases}
 a_{00} x_0 = b_0 \\
 a_{10} x_0 + a_{11} x_1 = b_1 \\
 a_{20} x_0 + a_{21} x_1 + a_{22} x_2 = b_2 \\
 a_{30} x_0 + a_{31} x_1 + a_{32} x_2 + a_{33} x_3 = b_3 \\
 a_{40} x_0 + a_{41} x_1 + a_{42} x_2 + a_{43} x_3 + a_{44} x_4 = b_4 \\
 a_{50} x_0 + a_{51} x_1 + a_{52} x_2 + a_{53} x_3 + a_{54} x_4 + a_{55} x_5 = b_5
 \end{cases}$$

$$\begin{cases}
 a_{22} x_2 = b_2 - (a_{20} x_0 + a_{21} x_1) = \tilde{b}_2 \\
 a_{32} x_2 + a_{33} x_3 = b_3 - (a_{30} x_0 + a_{31} x_1) = \tilde{b}_3
 \end{cases}$$

$$\tilde{B}_1 = B_1 - A_{10} X_0$$

Si modifica il termine noto $B_2 (b_4, b_5)$

$$\begin{cases}
 a_{00} x_0 = b_0 \\
 a_{10} x_0 + a_{11} x_1 = b_1 \\
 a_{20} x_0 + a_{21} x_1 + a_{22} x_2 = b_2 \\
 a_{30} x_0 + a_{31} x_1 + a_{32} x_2 + a_{33} x_3 = b_3 \\
 a_{40} x_0 + a_{41} x_1 + a_{42} x_2 + a_{43} x_3 + a_{44} x_4 = b_4 \\
 a_{50} x_0 + a_{51} x_1 + a_{52} x_2 + a_{53} x_3 + a_{54} x_4 + a_{55} x_5 = b_5
 \end{cases}$$

$$\begin{cases}
 a_{42} x_2 + a_{43} x_3 + a_{44} x_4 = b_4 - (a_{40} x_0 + a_{41} x_1) = \tilde{b}_4 \\
 a_{52} x_2 + a_{53} x_3 + a_{54} x_4 + a_{55} x_5 = b_5 - (a_{50} x_0 + a_{51} x_1) = \tilde{b}_5
 \end{cases}$$

$$\tilde{B}_2 = B_2 - A_{20} X_0$$

Si considera il sistema "attivo"

The diagram illustrates a linear system with a yellow matrix and a yellow vector b . The system is represented by four equations:

$$\begin{aligned} & a_{22} x_2 = b_2 \\ & a_{32} x_2 + a_{33} x_3 = b_3 \\ & a_{42} x_2 + a_{43} x_3 + a_{44} x_4 = b_4 \\ & a_{52} x_2 + a_{53} x_3 + a_{54} x_4 + a_{55} x_5 = b_5 \end{aligned}$$

A large watermark 'A' is overlaid on the diagram.

$$\begin{cases}
 a_{22} x_2 & = \tilde{b}_2 \\
 a_{32} x_2 + a_{33} x_3 & = \tilde{b}_3 \\
 a_{42} x_2 + a_{43} x_3 + a_{44} x_4 & = \tilde{b}_4 \\
 a_{52} x_2 + a_{53} x_3 + a_{54} x_4 + a_{55} x_5 & = \tilde{b}_5
 \end{cases}$$

Si Calcola $X_1(x_2, x_3)$
risolvendo il sistema
Triangolare Inferiore
 $A_{22}X_1=B_1$

Si modifica il termine noto B_2

$$\begin{cases}
 a_{22} x_2 \\
 a_{32} x_2 + a_{33} x_3 \\
 a_{42} x_2 + a_{43} x_3 + a_{44} x_4 \\
 a_{52} x_2 + a_{53} x_3 + a_{54} x_4 + a_{55} x_5
 \end{cases}
 =
 \begin{cases}
 \tilde{b}_2 \\
 \tilde{b}_3 \\
 \tilde{b}_4 \\
 \tilde{b}_5
 \end{cases}$$

$$\begin{cases}
 a_{44} x_4 = \tilde{b}_4 - (a_{42} x_2 + a_{43} x_3) = \hat{b}_4 \\
 a_{54} x_4 + a_{55} x_5 = \tilde{b}_5 - (a_{52} x_2 + a_{53} x_3) = \hat{b}_5
 \end{cases}$$

$$\underline{B_2 - A_{20}X_0 - A_{21}X_0 = \hat{B}_2}$$

$$\begin{cases} a_{44} x_4 = \hat{b}_4 \\ a_{54} x_4 + a_{55} x_5 = \hat{b}_5 \end{cases}$$

Si Calcola $X_2 = (x_4, x_5)$
risolvendo il sistema
Triangolare Inferiore

$$A_{33} X_2 = B_2$$

Passo 1: partizionamento a blocchi, risoluzione sistema $A_{00} X_0 = B_0$, aggiornamento

$a_{00} x_0$																				$=b_0$	
$a_{10} x_0 + a_{11} x_1$																					$=b_1$
$a_{20} x_0 + a_{21} x_1 + a_{22} x_2$																					$=b_2$
$a_{30} x_0 + a_{31} x_1 + a_{32} x_2 + a_{33} x_3$																					$=b_3$
$a_{40} x_0 + a_{41} x_1 + a_{42} x_2 + a_{43} x_3 + a_{44} x_4$																					$=b_4$
$a_{50} x_0 + a_{51} x_1 + a_{52} x_2 + a_{53} x_3 + a_{54} x_4 + a_{55} x_5$																					$=b_5$
$a_{60} x_0 + a_{61} x_1 + a_{62} x_2 + a_{63} x_3 + a_{64} x_4 + a_{65} x_5 + a_{66} x_6$																					$=b_6$
$a_{70} x_0 + a_{71} x_1 + a_{72} x_2 + a_{73} x_3 + a_{74} x_4 + a_{75} x_5 + a_{76} x_6 + a_{77} x_7$																					$=b_7$
$a_{80} x_0 + a_{81} x_1 + a_{82} x_2 + a_{83} x_3 + a_{84} x_4 + a_{85} x_5 + a_{86} x_6 + a_{87} x_7 + a_{88} x_8$																					$=b_8$
$a_{90} x_0 + a_{91} x_1 + a_{92} x_2 + a_{93} x_3 + a_{94} x_4 + a_{95} x_5 + a_{96} x_6 + a_{97} x_7 + a_{98} x_8 + a_{99} x_9$																					$=b_9$
$a_{100} x_0 + a_{101} x_1 + a_{102} x_2 + a_{103} x_3 + a_{104} x_4 + a_{105} x_5 + a_{106} x_6 + a_{107} x_7 + a_{108} x_8 + a_{109} x_9 + a_{1010} x_{10}$																					$=b_{10}$
$a_{110} x_0 + a_{111} x_1 + a_{112} x_2 + a_{113} x_3 + a_{114} x_4 + a_{115} x_5 + a_{116} x_6 + a_{117} x_7 + a_{118} x_8 + a_{119} x_9 + a_{1110} x_{10} + a_{1111} x_{11}$																					$=b_{11}$

Dopo il Passo 1

The diagram shows a linear system matrix with 11 rows and 11 columns. The matrix is partitioned into colored blocks: a light blue block at the top-left, an orange block at the top-middle, a light blue block at the top-right, and a light blue block at the bottom-left. The rightmost column is shaded light blue. A large watermark 'Maurilio' is overlaid on the matrix. The system is represented as follows:

$$\begin{array}{l}
 a_{22} x \\
 a_{32} x + a_{33} x \\
 a_{42} x + a_{43} x + a_{44} x \\
 a_{52} x + a_{53} x + a_{54} x + a_{55} x \\
 a_{62} x + a_{63} x + a_{64} x + a_{65} x + a_{66} x \\
 a_{72} x + a_{73} x + a_{74} x + a_{75} x + a_{76} x + a_{77} x \\
 a_{82} x + a_{83} x + a_{84} x + a_{85} x + a_{86} x + a_{87} x + a_{88} x \\
 a_{92} x + a_{93} x + a_{94} x + a_{95} x + a_{96} x + a_{97} x + a_{98} x + a_{99} x \\
 + a_{102} x + a_{103} x + a_{104} x + a_{105} x + a_{106} x + a_{107} x + a_{108} x + a_{109} x + a_{1010} x \\
 + a_{112} x + a_{113} x + a_{114} x + a_{115} x + a_{116} x + a_{117} x + a_{118} x + a_{119} x + a_{1110} x + a_{1111} x
 \end{array}
 =
 \begin{array}{l}
 \tilde{b}_2 \\
 \tilde{b}_3 \\
 \tilde{b}_4 \\
 \tilde{b}_5 \\
 \tilde{b}_6 \\
 \tilde{b}_7 \\
 \tilde{b}_8 \\
 \tilde{b}_9 \\
 \tilde{b}_{10} \\
 \tilde{b}_{11}
 \end{array}$$

Si ottiene il seguente Sistema "Attivo"

passo 2; partizionamento , risoluzione , aggiornamento...

Parte attiva del sistema triangolare

$a_{22} x$												$= \tilde{b}_2$
$a_{32} x + a_{33} x$												$= \tilde{b}_3$
$a_{42} x + a_{43} x + a_{44} x$												$= \tilde{b}_4$
$a_{52} x + a_{53} x + a_{54} x + a_{55} x$												$= \tilde{b}_5$
$a_{62} x + a_{63} x + a_{64} x + a_{65} x + a_{66} x$												$= \tilde{b}_6$
$a_{72} x + a_{73} x + a_{74} x + a_{75} x + a_{76} x + a_{77} x$												$= \tilde{b}_7$
$a_{82} x + a_{83} x + a_{84} x + a_{85} x + a_{86} x + a_{87} x + a_{88} x$												$= \tilde{b}_8$
$a_{92} x + a_{93} x + a_{94} x + a_{95} x + a_{96} x + a_{97} x + a_{98} x + a_{99} x$												$= \tilde{b}_9$
$a_{102} x + a_{103} x + a_{104} x + a_{105} x + a_{106} x + a_{107} x + a_{108} x + a_{109} x + a_{1010} x$												$= \tilde{b}_{10}$
$a_{112} x + a_{113} x + a_{114} x + a_{115} x + a_{116} x + a_{117} x + a_{118} x + a_{119} x + a_{1110} x + a_{1111} x$												$= \tilde{b}_{11}$

Dopo il Passo 2

The diagram shows a matrix system with 11 rows. The matrix is partitioned into blocks: a top orange block, a blue block, a green block, an orange block, and a blue block. The right-hand side vector \hat{b} is also shown with a corresponding orange block at the top. A large watermark 'MURILLO' is overlaid on the diagram.

$$\begin{array}{l}
 \left. \begin{array}{l}
 a_{44} x \\
 a_{54} x + a_{55} x \\
 a_{64} x + a_{65} x + a_{66} x \\
 a_{74} x + a_{75} x + a_{76} x + a_{77} x \\
 a_{84} x + a_{85} x + a_{86} x + a_{87} x + a_{88} x \\
 a_{94} x + a_{95} x + a_{96} x + a_{97} x + a_{98} x + a_{99} x \\
 + a_{104} x + a_{105} x + a_{106} x + a_{107} x + a_{108} x + a_{109} x + a_{110} x \\
 + a_{114} x + a_{115} x + a_{116} x + a_{117} x + a_{118} x + a_{119} x + a_{1110} x + a_{1111} x
 \end{array} \right\} = \hat{b}
 \end{array}$$

Si ottiene il seguente Sistema "Attivo"

Passo 3

Parte attiva del sistema triangolare

$$\begin{array}{l} a_{44} x_4 \\ a_{54} x_4 + a_{55} x_5 \\ a_{64} x_4 + a_{65} x_5 + a_{66} x_6 \\ a_{74} x_4 + a_{75} x_5 + a_{76} x_6 + a_{77} x_7 \\ a_{84} x_4 + a_{85} x_5 + a_{86} x_6 + a_{87} x_7 + a_{88} x_8 \\ a_{94} x_4 + a_{95} x_5 + a_{96} x_6 + a_{97} x_7 + a_{98} x_8 + a_{99} x_9 \\ a_{10,4} x_4 + a_{10,5} x_5 + a_{10,6} x_6 + a_{10,7} x_7 + a_{10,8} x_8 + a_{10,9} x_9 + a_{10,10} x_{10} \\ a_{11,4} x_4 + a_{11,5} x_5 + a_{11,6} x_6 + a_{11,7} x_7 + a_{11,8} x_8 + a_{11,9} x_9 + a_{11,10} x_{10} + a_{11,11} x_{11} \end{array} \quad \begin{array}{l} = \hat{b}_4 \\ = \hat{b}_5 \\ = \hat{b}_6 \\ = \hat{b}_7 \\ = \hat{b}_8 \\ = \hat{b}_9 \\ = \hat{b}_{10} \\ = \hat{b}_{11} \end{array}$$

Dopo il passo 3...

The diagram shows a matrix system with four rows. The first and last rows are highlighted with blue hatching. The second row is light blue, and the third row is orange. A large, stylized watermark 'MURILLO' is overlaid on the equations. The equations are as follows:

$$\begin{array}{l} a_{66} x_6 \\ a_{76} x_6 + a_{77} x_7 \\ a_{86} x_6 + a_{87} x_7 + a_{88} x_8 \\ a_{96} x_6 + a_{97} x_7 + a_{98} x_8 + a_{99} x_9 \\ + a_{10,6} x_6 + a_{10,7} x_7 + a_{10,8} x_8 + a_{10,9} x_9 + a_{10,10} x_{10} \\ + a_{11,6} x_6 + a_{11,7} x_7 + a_{11,8} x_8 + a_{11,9} x_9 + a_{11,10} x_{10} + a_{11,11} x_{11} \end{array} = \begin{array}{l} \ddot{b}_6 \\ \ddot{b}_7 \\ \ddot{b}_8 \\ \ddot{b}_9 \\ \ddot{b}_{10} \\ \ddot{b}_{11} \end{array}$$

Si ottiene il seguente Sistema "Attivo"

Passo 4

Parte attiva del sistema triangolare

$$\begin{cases} a_{66} x_6 & = \ddot{b}_6 \\ a_{76} x_6 + a_{77} x_7 & = \ddot{b}_7 \\ a_{86} x_6 + a_{87} x_7 + a_{88} x_8 & = \ddot{b}_8 \\ a_{96} x_6 + a_{97} x_7 + a_{98} x_8 + a_{99} x_9 & = \ddot{b}_9 \\ a_{10,6} x_6 + a_{10,7} x_7 + a_{10,8} x_8 + a_{10,9} x_9 + a_{10,10} x_{10} & = \ddot{b}_{10} \\ a_{11,6} x_6 + a_{11,7} x_7 + a_{11,8} x_8 + a_{11,9} x_9 + a_{11,10} x_{10} + a_{11,11} x_{11} & = \ddot{b}_{11} \end{cases}$$

Dopo il Passo 4

$$\begin{array}{l}
 a_{88} x_8 = \bar{b}_8 \\
 a_{98} x_8 + a_{99} x_9 = \bar{b}_9 \\
 + a_{10,8} x_8 + a_{10,9} x_9 + a_{10,10} x_{10} = \bar{b}_{10} \\
 + a_{11,8} x_8 + a_{11,9} x_9 + a_{11,10} x_{10} + a_{11,11} x_{11} = \bar{b}_{11}
 \end{array}$$

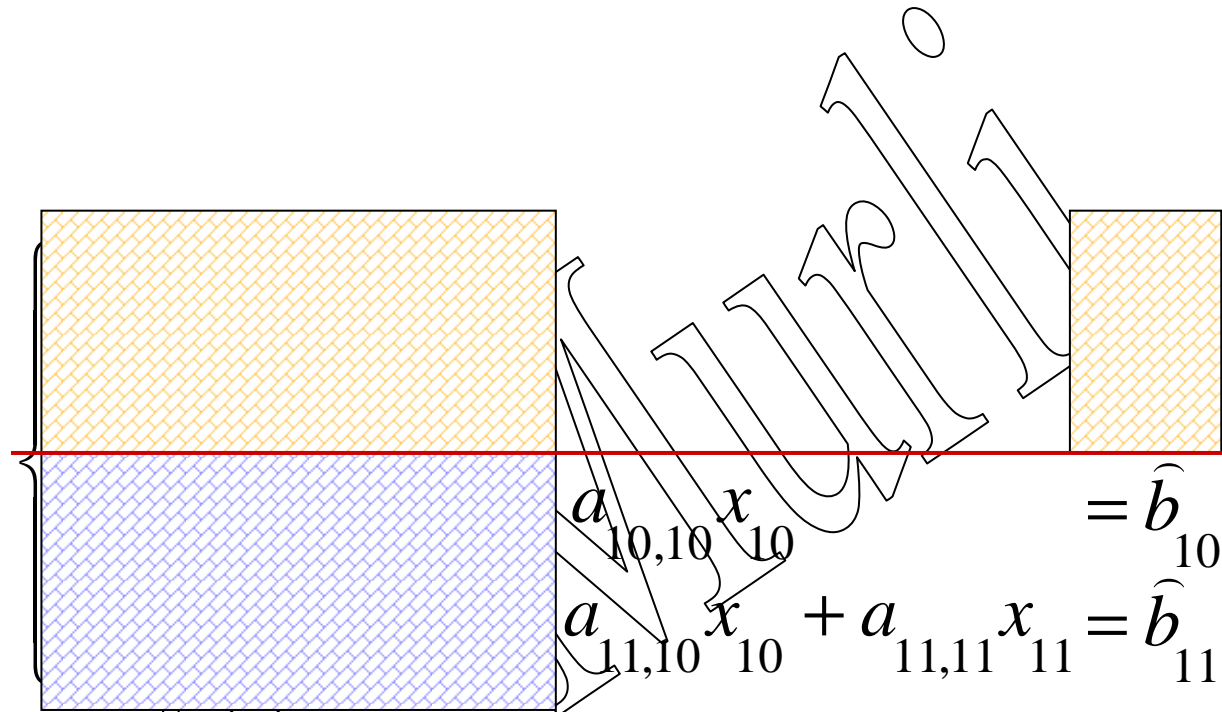
Si ottiene il seguente Sistema "Attivo"

Parte attiva del sistema triangolare

$$\begin{array}{l}
 a_{88} x_8 \\
 a_{98} x_8 + a_{99} x_9 \\
 \hline
 a_{10,8} x_8 + a_{10,9} x_9 + a_{10,10} x_{10} \\
 a_{11,8} x_8 + a_{11,9} x_9 + a_{11,10} x_{10} + a_{11,11} x_{11}
 \end{array}
 =
 \begin{array}{l}
 \bar{b}_8 \\
 \bar{b}_9 \\
 \bar{b}_{10} \\
 \bar{b}_{11}
 \end{array}$$

risolve il sistema $\mathbf{A}_{44}\mathbf{X}_4 = \mathbf{B}_4$

Dopo il passo 5...



Si ottiene il seguente Sistema "Attivo"

Passo 6 (=12 / 2)

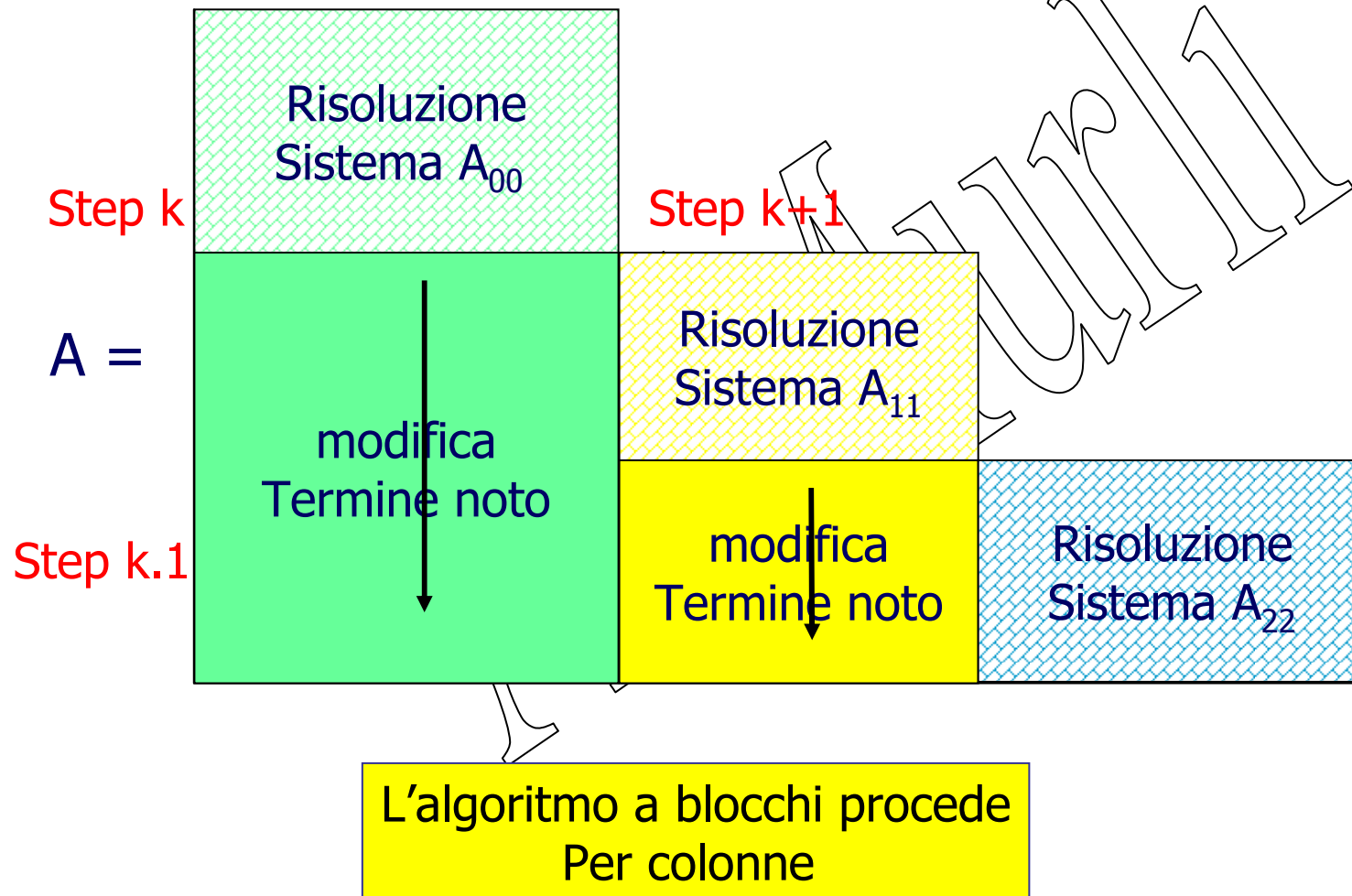
Parte attiva del sistema triangolare

$$\begin{cases} a_{10,10} x_{10} = \hat{b}_{10} \\ a_{11,10} x_{10} + a_{11,11} x_{11} = \hat{b}_{11} \end{cases}$$

In generale i passi dell'algoritmo a blocchi sono
 $n = N/nb$

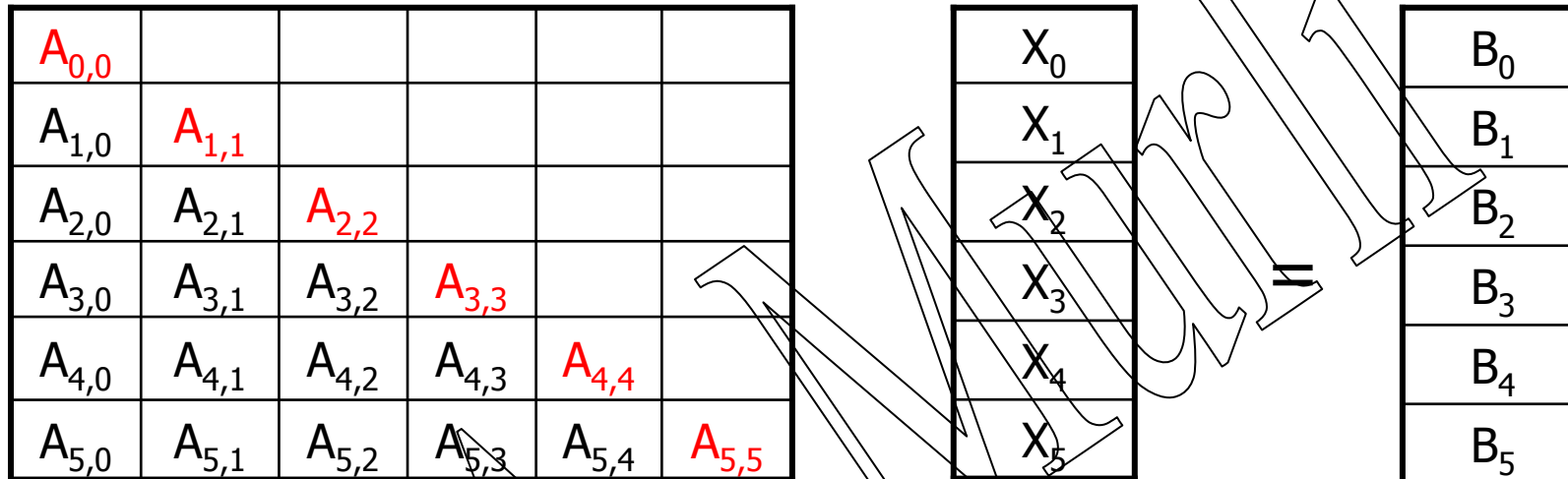
Algoritmo a blocchi: Fasi di calcolo

La matrice è decomposta in 3 x 3 blocchi



Algoritmo a blocchi

Partizioniamo la matrice , il vettore x e il vettore b in blocchi



Le matrici A_{ji} sono matrici **triangolari inferiori**

Algoritmo a blocchi

$$\begin{cases} A_{00}x_0 & = B_0 \\ A_{10}x_0 + A_{11}x_1 & = B_1 \\ A_{20}x_0 + A_{21}x_1 + A_{22}x_2 & = B_2 \\ \vdots & \vdots \\ A_{n0}x_0 + A_{n1}x_1 + A_{n2}x_2 + \dots + A_{nn}x_n & = B_n \end{cases}$$

Passo 0: $A_{00} X_0 = B_0$

Passo 1: $B_1 = A_{10} X_0 + A_{11} X_1$

Passo 2: $B_2 = A_{20} X_0 + A_{21} X_1 + A_{22} X_2$

Passo 3: $B_3 = A_{30} X_0 + A_{31} X_1 + A_{32} X_2 + A_{33} X_3$

Passo n: $B_n = A_{n0} X_0 + A_{n1} X_1 + A_{n2} X_2 + \dots + A_{nn} X_n$

Algoritmo a blocchi: sequenza temporale ...

$$\begin{cases}
 A_{00}x_0 & & & & & = B_0 \\
 A_{10}x_0 + A_{11}x_1 & & & & & = B_1 \\
 A_{20}x_0 + A_{21}x_1 + A_{22}x_2 & & & & & = B_2 \\
 \vdots & \mathbf{0.1} & & & & \vdots \\
 A_{n0}x_0 + A_{n1}x_1 + A_{n2}x_2 + \dots + A_{nn}x_n & & & & & = B_n
 \end{cases}$$

The diagram shows a system of linear equations. The first row is $A_{00}x_0 = B_0$. The second row is $A_{10}x_0 + A_{11}x_1 = B_1$. The third row is $A_{20}x_0 + A_{21}x_1 + A_{22}x_2 = B_2$. The fourth row is \vdots . The fifth row is $A_{n0}x_0 + A_{n1}x_1 + A_{n2}x_2 + \dots + A_{nn}x_n = B_n$. Red boxes highlight $A_{00}x_0$, $A_{11}x_1$, $A_{22}x_2$, and $A_{nn}x_n$. Red labels **0**, **1**, **2**, and **2.1** are placed above the corresponding terms. A blue box highlights the first column of coefficients. A blue box highlights the right-hand side terms $B_0, B_1, B_2, \dots, B_n$. A blue box highlights the diagonal elements $A_{00}, A_{11}, A_{22}, \dots, A_{nn}$. A blue box highlights the sub-diagonal elements $A_{10}, A_{20}, \dots, A_{n0}$. A blue box highlights the sub-diagonal elements $A_{21}, A_{31}, \dots, A_{n1}$. A blue box highlights the sub-diagonal elements $A_{32}, A_{42}, \dots, A_{n2}$. A blue box highlights the sub-diagonal elements $A_{43}, A_{53}, \dots, A_{n3}$. A blue box highlights the sub-diagonal elements $A_{54}, A_{64}, \dots, A_{n4}$. A blue box highlights the sub-diagonal elements $A_{65}, A_{75}, \dots, A_{n5}$. A blue box highlights the sub-diagonal elements $A_{76}, A_{86}, \dots, A_{n6}$. A blue box highlights the sub-diagonal elements $A_{87}, A_{97}, \dots, A_{n7}$. A blue box highlights the sub-diagonal elements $A_{98}, A_{108}, \dots, A_{n8}$. A blue box highlights the sub-diagonal elements $A_{109}, A_{119}, \dots, A_{n9}$. A blue box highlights the sub-diagonal elements $A_{1110}, A_{1210}, \dots, A_{n10}$. A blue box highlights the sub-diagonal elements $A_{1211}, A_{1311}, \dots, A_{n11}$. A blue box highlights the sub-diagonal elements $A_{1312}, A_{1412}, \dots, A_{n12}$. A blue box highlights the sub-diagonal elements $A_{1413}, A_{1513}, \dots, A_{n13}$. A blue box highlights the sub-diagonal elements $A_{1514}, A_{1614}, \dots, A_{n14}$. A blue box highlights the sub-diagonal elements $A_{1615}, A_{1715}, \dots, A_{n15}$. A blue box highlights the sub-diagonal elements $A_{1716}, A_{1816}, \dots, A_{n16}$. A blue box highlights the sub-diagonal elements $A_{1817}, A_{1917}, \dots, A_{n17}$. A blue box highlights the sub-diagonal elements $A_{1918}, A_{2018}, \dots, A_{n18}$. A blue box highlights the sub-diagonal elements $A_{2019}, A_{2119}, \dots, A_{n19}$. A blue box highlights the sub-diagonal elements $A_{2120}, A_{2220}, \dots, A_{n20}$. A blue box highlights the sub-diagonal elements $A_{2221}, A_{2321}, \dots, A_{n21}$. A blue box highlights the sub-diagonal elements $A_{2322}, A_{2422}, \dots, A_{n22}$. A blue box highlights the sub-diagonal elements $A_{2423}, A_{2523}, \dots, A_{n23}$. A blue box highlights the sub-diagonal elements $A_{2524}, A_{2624}, \dots, A_{n24}$. A blue box highlights the sub-diagonal elements $A_{2625}, A_{2725}, \dots, A_{n25}$. A blue box highlights the sub-diagonal elements $A_{2726}, A_{2826}, \dots, A_{n26}$. A blue box highlights the sub-diagonal elements $A_{2827}, A_{2927}, \dots, A_{n27}$. A blue box highlights the sub-diagonal elements $A_{2928}, A_{3028}, \dots, A_{n28}$. A blue box highlights the sub-diagonal elements $A_{3029}, A_{3129}, \dots, A_{n29}$. A blue box highlights the sub-diagonal elements $A_{3130}, A_{3230}, \dots, A_{n30}$. A blue box highlights the sub-diagonal elements $A_{3231}, A_{3331}, \dots, A_{n31}$. A blue box highlights the sub-diagonal elements $A_{3332}, A_{3432}, \dots, A_{n32}$. A blue box highlights the sub-diagonal elements $A_{3433}, A_{3533}, \dots, A_{n33}$. A blue box highlights the sub-diagonal elements $A_{3534}, A_{3634}, \dots, A_{n34}$. A blue box highlights the sub-diagonal elements $A_{3635}, A_{3735}, \dots, A_{n35}$. A blue box highlights the sub-diagonal elements $A_{3736}, A_{3836}, \dots, A_{n36}$. A blue box highlights the sub-diagonal elements $A_{3837}, A_{3937}, \dots, A_{n37}$. A blue box highlights the sub-diagonal elements $A_{3938}, A_{4038}, \dots, A_{n38}$. A blue box highlights the sub-diagonal elements $A_{4039}, A_{4139}, \dots, A_{n39}$. A blue box highlights the sub-diagonal elements $A_{4140}, A_{4240}, \dots, A_{n40}$. A blue box highlights the sub-diagonal elements $A_{4241}, A_{4341}, \dots, A_{n41}$. A blue box highlights the sub-diagonal elements $A_{4342}, A_{4442}, \dots, A_{n42}$. A blue box highlights the sub-diagonal elements $A_{4443}, A_{4543}, \dots, A_{n43}$. A blue box highlights the sub-diagonal elements $A_{4544}, A_{4644}, \dots, A_{n44}$. A blue box highlights the sub-diagonal elements $A_{4645}, A_{4745}, \dots, A_{n45}$. A blue box highlights the sub-diagonal elements $A_{4746}, A_{4846}, \dots, A_{n46}$. A blue box highlights the sub-diagonal elements $A_{4847}, A_{4947}, \dots, A_{n47}$. A blue box highlights the sub-diagonal elements $A_{4948}, A_{5048}, \dots, A_{n48}$. A blue box highlights the sub-diagonal elements $A_{5049}, A_{5149}, \dots, A_{n49}$. A blue box highlights the sub-diagonal elements $A_{5150}, A_{5250}, \dots, A_{n50}$. A blue box highlights the sub-diagonal elements $A_{5251}, A_{5351}, \dots, A_{n51}$. A blue box highlights the sub-diagonal elements $A_{5352}, A_{5452}, \dots, A_{n52}$. A blue box highlights the sub-diagonal elements $A_{5453}, A_{5553}, \dots, A_{n53}$. A blue box highlights the sub-diagonal elements $A_{5554}, A_{5654}, \dots, A_{n54}$. A blue box highlights the sub-diagonal elements $A_{5655}, A_{5755}, \dots, A_{n55}$. A blue box highlights the sub-diagonal elements $A_{5756}, A_{5856}, \dots, A_{n56}$. A blue box highlights the sub-diagonal elements $A_{5857}, A_{5957}, \dots, A_{n57}$. A blue box highlights the sub-diagonal elements $A_{5958}, A_{6058}, \dots, A_{n58}$. A blue box highlights the sub-diagonal elements $A_{6059}, A_{6159}, \dots, A_{n59}$. A blue box highlights the sub-diagonal elements $A_{6160}, A_{6260}, \dots, A_{n60}$. A blue box highlights the sub-diagonal elements $A_{6261}, A_{6361}, \dots, A_{n61}$. A blue box highlights the sub-diagonal elements $A_{6362}, A_{6462}, \dots, A_{n62}$. A blue box highlights the sub-diagonal elements $A_{6463}, A_{6563}, \dots, A_{n63}$. A blue box highlights the sub-diagonal elements $A_{6564}, A_{6664}, \dots, A_{n64}$. A blue box highlights the sub-diagonal elements $A_{6665}, A_{6765}, \dots, A_{n65}$. A blue box highlights the sub-diagonal elements $A_{6766}, A_{6866}, \dots, A_{n66}$. A blue box highlights the sub-diagonal elements $A_{6867}, A_{6967}, \dots, A_{n67}$. A blue box highlights the sub-diagonal elements $A_{6968}, A_{7068}, \dots, A_{n68}$. A blue box highlights the sub-diagonal elements $A_{7069}, A_{7169}, \dots, A_{n69}$. A blue box highlights the sub-diagonal elements $A_{7170}, A_{7270}, \dots, A_{n70}$. A blue box highlights the sub-diagonal elements $A_{7271}, A_{7371}, \dots, A_{n71}$. A blue box highlights the sub-diagonal elements $A_{7372}, A_{7472}, \dots, A_{n72}$. A blue box highlights the sub-diagonal elements $A_{7473}, A_{7573}, \dots, A_{n73}$. A blue box highlights the sub-diagonal elements $A_{7574}, A_{7674}, \dots, A_{n74}$. A blue box highlights the sub-diagonal elements $A_{7675}, A_{7775}, \dots, A_{n75}$. A blue box highlights the sub-diagonal elements $A_{7776}, A_{7876}, \dots, A_{n76}$. A blue box highlights the sub-diagonal elements $A_{7877}, A_{7977}, \dots, A_{n77}$. A blue box highlights the sub-diagonal elements $A_{7978}, A_{8078}, \dots, A_{n78}$. A blue box highlights the sub-diagonal elements $A_{8079}, A_{8179}, \dots, A_{n79}$. A blue box highlights the sub-diagonal elements $A_{8180}, A_{8280}, \dots, A_{n80}$. A blue box highlights the sub-diagonal elements $A_{8281}, A_{8381}, \dots, A_{n81}$. A blue box highlights the sub-diagonal elements $A_{8382}, A_{8482}, \dots, A_{n82}$. A blue box highlights the sub-diagonal elements $A_{8483}, A_{8583}, \dots, A_{n83}$. A blue box highlights the sub-diagonal elements $A_{8584}, A_{8684}, \dots, A_{n84}$. A blue box highlights the sub-diagonal elements $A_{8685}, A_{8785}, \dots, A_{n85}$. A blue box highlights the sub-diagonal elements $A_{8786}, A_{8886}, \dots, A_{n86}$. A blue box highlights the sub-diagonal elements $A_{8887}, A_{8987}, \dots, A_{n87}$. A blue box highlights the sub-diagonal elements $A_{8988}, A_{9088}, \dots, A_{n88}$. A blue box highlights the sub-diagonal elements $A_{9089}, A_{9189}, \dots, A_{n89}$. A blue box highlights the sub-diagonal elements $A_{9190}, A_{9290}, \dots, A_{n90}$. A blue box highlights the sub-diagonal elements $A_{9291}, A_{9391}, \dots, A_{n91}$. A blue box highlights the sub-diagonal elements $A_{9392}, A_{9492}, \dots, A_{n92}$. A blue box highlights the sub-diagonal elements $A_{9493}, A_{9593}, \dots, A_{n93}$. A blue box highlights the sub-diagonal elements $A_{9594}, A_{9694}, \dots, A_{n94}$. A blue box highlights the sub-diagonal elements $A_{9695}, A_{9795}, \dots, A_{n95}$. A blue box highlights the sub-diagonal elements $A_{9796}, A_{9896}, \dots, A_{n96}$. A blue box highlights the sub-diagonal elements $A_{9897}, A_{9997}, \dots, A_{n97}$. A blue box highlights the sub-diagonal elements $A_{9998}, A_{10098}, \dots, A_{n98}$. A blue box highlights the sub-diagonal elements $A_{10099}, A_{10199}, \dots, A_{n99}$. A blue box highlights the sub-diagonal elements $A_{101100}, A_{102100}, \dots, A_{n100}$.

Step k: Risoluzione sistema triangolare $A_{kk} X_k = B_k$

Step k.1: Modifica termine noto del sistema relativo alla matrice A_{k+1k+1}

Step k+1: Risoluzione sist. triangolare
 $A_{k+1k+1} X_{k+1} = B_{k+1}$

Fine lezione

A.M. 11