Calcolo Scientifico

a.a. 2007-2008

Problema …

Sistemi triangolari

Sistemi triangolari **inferiori**

$$\begin{cases} a_{00}x_0 & = b_0 \\ a_{10}x_0 + a_{11}x_1 & = b_1 \\ a_{20}x_0 + a_{21}x_1 + a_{22}x_2 & = b_2 \end{cases}$$

Forward Substitution

(x_0 , x_1 , x_2)

Sistemi triangolari **superiori**

$$\begin{cases} a_{00}x_0 + a_{01}x_1 + a_{02}x_2 = b_0 \\ a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{22}x_2 = b_2 \end{cases}$$

Backward Substitution

(x_2 , x_1 , x_0)

Sistemi triangolari inferiori …

$$\begin{cases} a_{00}x_0 & = b_0 \\ a_{10}x_0 + a_{11}x_1 & = b_1 \\ a_{20}x_0 + a_{21}x_1 + a_{22}x_2 & = b_2 \\ \vdots \quad \vdots \quad \vdots \quad \ddots & = \vdots \\ a_{n0}x_0 + a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n & = b_n \end{cases}$$

Algoritmo di Forward Substitution

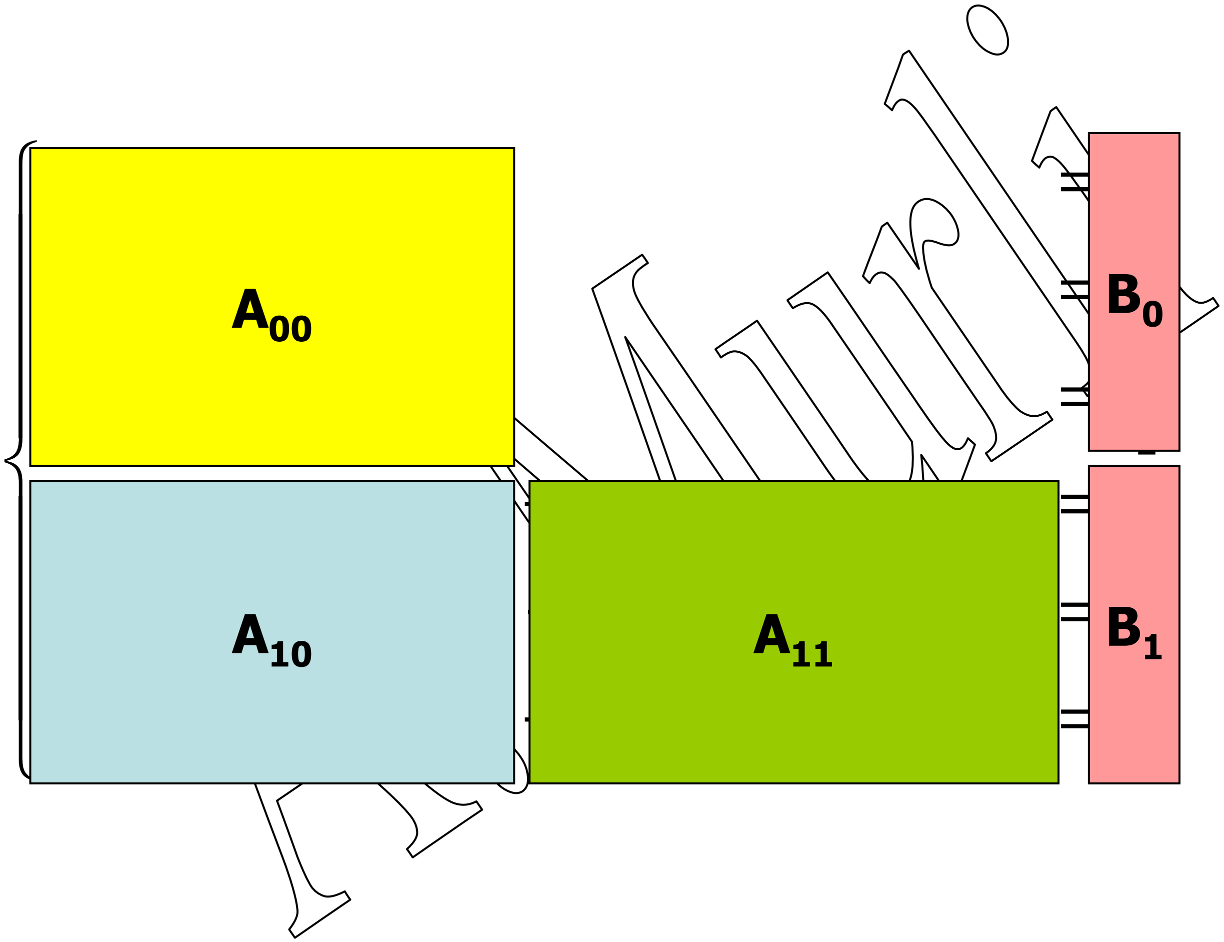
$$\begin{cases} X_0 = b_0 / a_{00} \\ x_i = \left(b_i - \sum_{j=0}^{i-1} a_{ij} x_j \right) \Big/ a_{ii} \end{cases}$$

$\forall$ i= 1, … , n

Consideriamo un sistema di dimensione n=6 e 2 blocchi 3x3

$$
\begin{cases}
a_{00}x_0 = b_0 \\
a_{10}x_0 + a_{11}x_1 = b_1 \\
a_{20}x_0 + a_{21}x_1 + a_{22}x_2 = b_2 \\
a_{30}x_0 + a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \\
a_{40}x_0 + a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 = b_4 \\
a_{50}x_0 + a_{51}x_1 + a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 = b_5
\end{cases}
$$

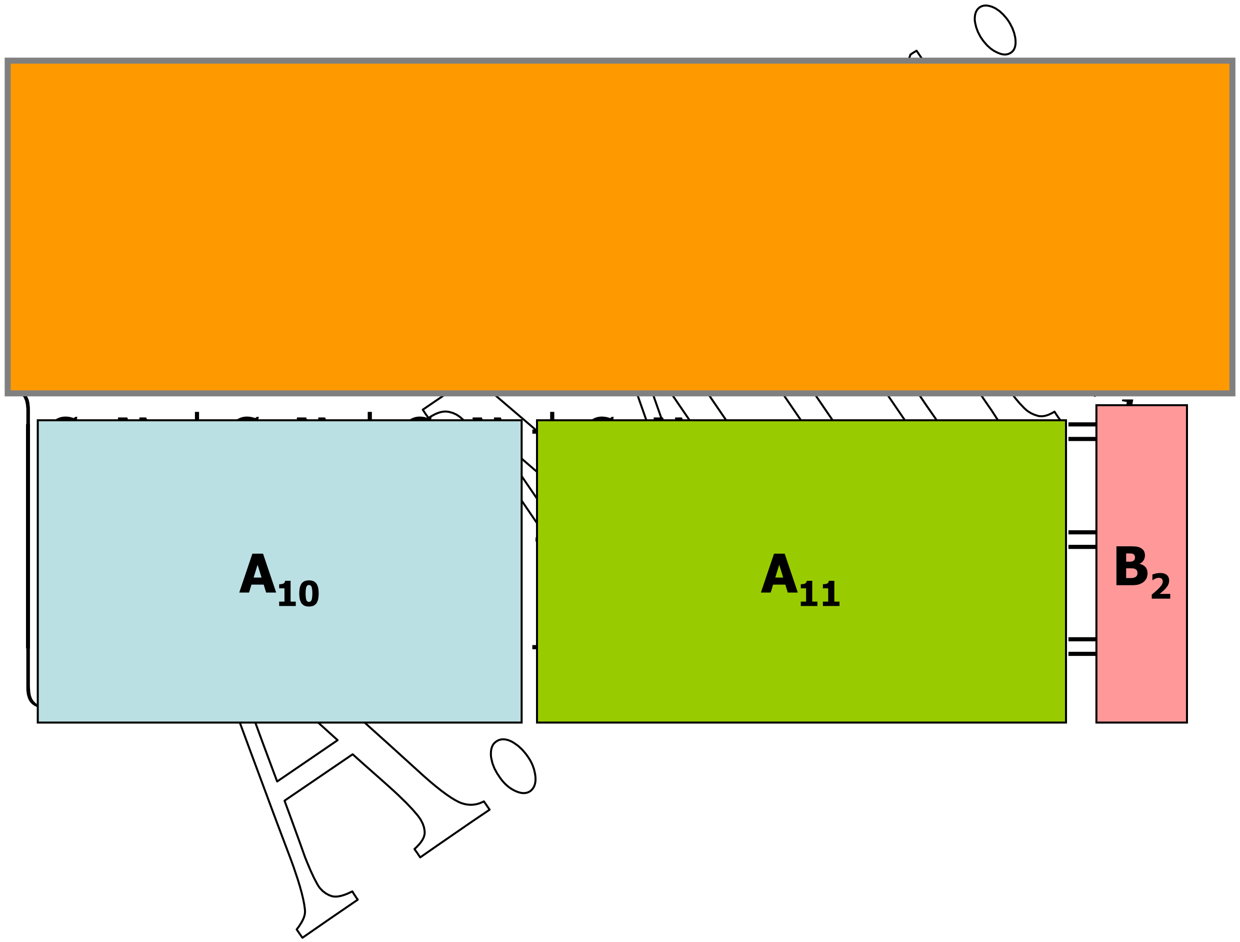
Partizioniamo la matrice a blocchi di dimensione 3 x3



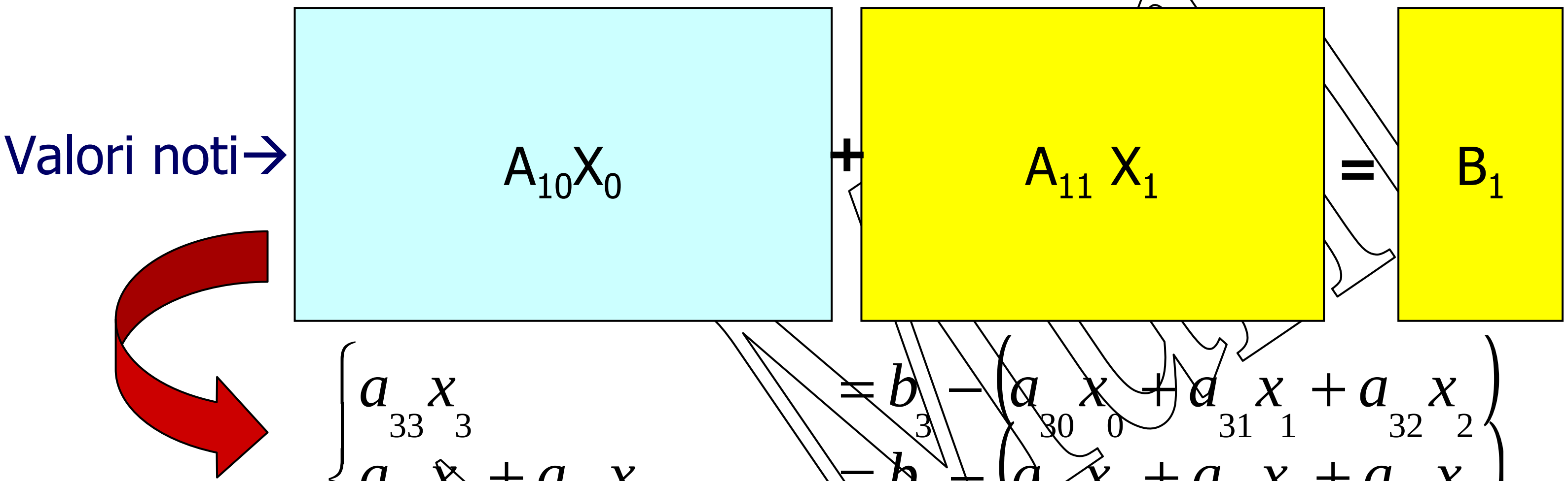
Si risolve il sistema $A_{00}X_0 = B_0$

$$
\begin{cases}
a_{00}x_0 = b_0 \\
a_{10}x_0 + a_{11}x_1 = b_1 \\
a_{20}x_0 + a_{21}x_1 + a_{22}x_2 = b_2 \\
a_{30}x_0 + a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \\
a_{40}x_0 + a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 = b_4 \\
a_{50}x_0 + a_{51}x_1 + a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 = b_5
\end{cases}
$$

Si considerano i blocchi rimanenti $A_{10}X_0 + A_{11} X_1 = B_1$



Dopo aver calcolato X_0 Si modifica il termine noto B_1



$$\begin{cases} a_{33}x_3 = b_3 - \left(a_{30}x_0 + a_{31}x_1 + a_{32}x_2\right) \\ a_{43}x_3 + a_{44}x_4 = b_4 - \left(a_{40}x_0 + a_{41}x_1 + a_{42}x_2\right) \\ a_{53}x_3 + a_{54}x_4 + a_{55}x_5 = b_5 - \left(a_{50}x_0 + a_{51}x_1 + a_{52}x_2\right) \end{cases}$$

$$A_{11}X_1 = \tilde{B}_1 \qquad B_1 - A_{10}X_0 = \tilde{B}_1$$

Consideriamo un sistema di dimensione n=6 e 3 blocchi 2x2

$$
\begin{cases}
a_{00}x_0 = b_0 \\
a_{10}x_0 + a_{11}x_1 = b_1 \\
a_{20}x_0 + a_{21}x_1 + a_{22}x_2 = b_2 \\
a_{30}x_0 + a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \\
a_{40}x_0 + a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 = b_4 \\
a_{50}x_0 + a_{51}x_1 + a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 = b_5
\end{cases}
$$

Si risolve il sistema $A_{00} X_0 = B_0$

$$
\begin{cases}
a_{00}x_0 & = b_0 \\
a_{10}x_0 + a_{11}x_1 & = b_1 \\
a_{20}x_0 + a_{21}x_1 + a_{22}x_2 & = b_2 \\
a_{30}x_0 + a_{31}x_1 + a_{32}x_2 + a_{33}x_3 & = b_3 \\
a_{40}x_0 + a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 & = b_4 \\
a_{50}x_0 + a_{51}x_1 + a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 & = b_5
\end{cases}
$$

Si modifica il termine noto B_1 (b_2, b_3)

$$\begin{cases}
a_{00}x_0 & = b_0 \\
a_{10}x_0 + a_{11}x_1 & = b_1 \\
a_{20}x_0 + a_{21}x_1 + a_{22}x_2 & = b_2 \\
a_{30}x_0 + a_{31}x_1 + a_{32}x_2 + a_{33}x_3 & = b_3 \\
a_{40}x_0 + a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 & = b_4 \\
a_{50}x_0 + a_{51}x_1 + a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 & = b_5
\end{cases}$$

$$\begin{cases}
a_{22}x_2 = b_2 - (a_{20}x_0 + a_{21}x_1) = \tilde{b}_2 \\
a_{32}x_2 + a_{33}x_3 = b_3 - (a_{30}x_0 + a_{31}x_1) = \tilde{b}_3
\end{cases}$$

$$\tilde{B}_1 = B_1 - A_{10}X_0$$

Si modifica il termine noto B_2 (b_4, b_5)

$$
\begin{cases}
a_{00}x_0 & = b_0 \\
a_{10}x_0 + a_{11}x_1 & = b_1 \\
a_{20}x_0 + a_{21}x_1 + a_{22}x_2 & = b_2 \\
a_{30}x_0 + a_{31}x_1 + a_{32}x_2 + a_{33}x_3 & = b_3 \\
a_{40}x_0 + a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 & = b_4 \\
a_{50}x_0 + a_{51}x_1 + a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 & = b_5
\end{cases}
$$

$$
\begin{cases}
a_{42}x_2 + a_{43}x_3 + a_{44}x_4 = b_4 - \left(a_{40}x_0 + a_{41}x_1\right) = \tilde{b}_4 \\
a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 = b_5 - \left(a_{50}x_0 + a_{51}x_1\right) = \tilde{b}_5
\end{cases}
$$

$$\tilde{B}_2 = B_2 - A_{20}X_0$$

Si considera il sistema "attivo"

$$\begin{cases} \quad + a_{22}x_2 = b_2 \\ \quad + a_{32}x_2 + a_{33}x_3 = b_3 \\ \quad + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 = b_4 \\ \quad + a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 = b_5 \end{cases}$$

$$
\begin{cases}
a_{22}x_2 & = \tilde{b}_2 \\
a_{32}x_2 + a_{33}x_3 & = \tilde{b}_3 \\
a_{42}x_2 + a_{43}x_3 + a_{44}x_4 & = \tilde{b}_4 \\
a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 & = \tilde{b}_5
\end{cases}
$$

Si Calcola X $_1$(x$_2$, x$_3$)
risolvendo il sistema
Triangolare Inferiore
$A_{22}X_1=B_1$

Si modifica il termine noto B_2

$$
\begin{cases}
a_{22}x_2 = \tilde{b}_2 \\
a_{32}x_2 + a_{33}x_3 = \tilde{b}_3 \\
a_{42}x_2 + a_{43}x_3 + a_{44}x_4 = \tilde{b}_4 \\
a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 = \tilde{b}_5
\end{cases}
$$

$$
\begin{cases}
a_{44}x_4 = \tilde{b}_4 - \left(a_{42}x_2 + a_{43}x_3\right) = \hat{b}_4 \\
a_{54}x_4 + a_{55}x_5 = \tilde{b}_5 - \left(a_{52}x_2 + a_{53}x_3\right) = \hat{b}_5
\end{cases}
$$

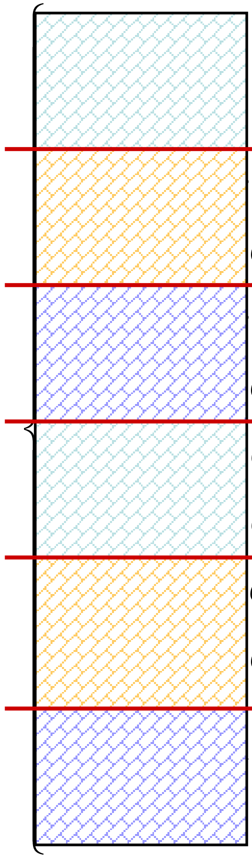
$$\underline{B_2 - A_{20}X_0} - A_{21}X_0 = \hat{B}_2$$

$$\begin{cases} a_{44}x_4 = \hat{b}_4 \\ a_{54}x_4 + a_{55}x_5 = \hat{b}_5 \end{cases}$$

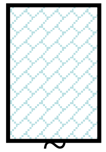
Si Calcola X_2 =(x_4, x_5)
risolvendo il sistema
Triangolare Inferiore
A_{33} X_2= B_2

Passo 1: partizionamento a blocchi, risoluzione sistema $A_{00}\ X_0=B_0$, aggiornamento

$$
\begin{cases}
a_{00}x_0 = b_0\\
a_{10}x_0 + a_{11}x_1 = b_1\\
a_{20}x_0 + a_{21}x_1 + a_{22}x_2 = b_2\\
a_{30}x_0 + a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3\\
a_{40}x_0 + a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 = b_4\\
a_{50}x_0 + a_{51}x_1 + a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 = b_5\\
a_{60}x_0 + a_{61}x_1 + a_{62}x_2 + a_{63}x_3 + a_{64}x_4 + a_{65}x_5 + a_{66}x_6 = b_6\\
a_{70}x_0 + a_{71}x_1 + a_{72}x_2 + a_{73}x_3 + a_{74}x_4 + a_{75}x_5 + a_{76}x_6 + a_{77}x_7 = b_7\\
a_{80}x_0 + a_{81}x_1 + a_{82}x_2 + a_{83}x_3 + a_{84}x_4 + a_{85}x_5 + a_{86}x_6 + a_{87}x_7 + a_{88}x_8 = b_8\\
a_{90}x_0 + a_{91}x_1 + a_{92}x_2 + a_{93}x_3 + a_{94}x_4 + a_{95}x_5 + a_{96}x_6 + a_{97}x_7 + a_{98}x_8 + a_{99}x_9 = b_9\\
a_{10,0}x_0 + a_{10,1}x_1 + a_{10,2}x_2 + a_{10,3}x_3 + a_{10,4}x_4 + a_{10,5}x_5 + a_{10,6}x_6 + a_{10,7}x_7 + a_{10,8}x_8 + a_{10,9}x_9 + a_{10,10}x_{10} = b_{10}\\
a_{11,0}x_0 + a_{11,1}x_1 + a_{11,2}x_2 + a_{11,3}x_3 + a_{11,4}x_4 + a_{11,5}x_5 + a_{11,6}x_6 + a_{11,7}x_7 + a_{11,8}x_8 + a_{11,9}x_9 + a_{11,10}x_{10} + a_{11,11}x_{11} = b_{11}
\end{cases}
$$



$$
\begin{cases}
a_{22}x_2 = \tilde{b}_2 \\
a_{32}x_2 + a_{33}x_3 = \tilde{b}_3 \\
a_{42}x_2 + a_{43}x_3 + a_{44}x_4 = \tilde{b}_4 \\
a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 = \tilde{b}_5 \\
a_{62}x_2 + a_{63}x_3 + a_{64}x_4 + a_{65}x_5 + a_{66}x_6 = \tilde{b}_6 \\
a_{72}x_2 + a_{73}x_3 + a_{74}x_4 + a_{75}x_5 + a_{76}x_6 + a_{77}x_7 = \tilde{b}_7 \\
a_{82}x_2 + a_{83}x_3 + a_{84}x_4 + a_{85}x_5 + a_{86}x_6 + a_{87}x_7 + a_{88}x_8 = \tilde{b}_8 \\
a_{92}x_2 + a_{93}x_3 + a_{94}x_4 + a_{95}x_5 + a_{96}x_6 + a_{97}x_7 + a_{98}x_8 + a_{99}x_9 = \tilde{b}_9 \\
+a_{10,2}x_2 + a_{10,3}x_3 + a_{10,4}x_4 + a_{10,5}x_5 + a_{10,6}x_6 + a_{10,7}x_7 + a_{10,8}x_8 + a_{10,9}x_9 + a_{10,10}x_{10} = \tilde{b}_{10} \\
+a_{11,2}x_2 + a_{11,3}x_3 + a_{11,4}x_4 + a_{11,5}x_5 + a_{11,6}x_6 + a_{11,7}x_7 + a_{11,8}x_8 + a_{11,9}x_9 + a_{11,10}x_{10} + a_{11,11}x_{11} = \tilde{b}_{11}
\end{cases}
$$



Si ottiene il seguente Sistema “Attivo”

passo 2; partizionamento , risoluzione , aggiornamento…

Parte attiva del sistema triangolare

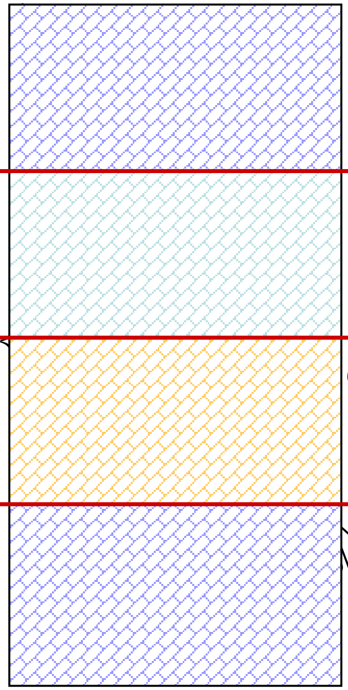
$$
\begin{cases}
a_{22}x_2 = \tilde{b}_2 \\
a_{32}x_2 + a_{33}x_3 = \tilde{b}_3 \\
a_{42}x_2 + a_{43}x_3 + a_{44}x_4 = \tilde{b}_4 \\
a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 = \tilde{b}_5 \\
a_{62}x_2 + a_{63}x_3 + a_{64}x_4 + a_{65}x_5 + a_{66}x_6 = \tilde{b}_6 \\
a_{72}x_2 + a_{73}x_3 + a_{74}x_4 + a_{75}x_5 + a_{76}x_6 + a_{77}x_7 = \tilde{b}_7 \\
a_{82}x_2 + a_{83}x_3 + a_{84}x_4 + a_{85}x_5 + a_{86}x_6 + a_{87}x_7 + a_{88}x_8 = \tilde{b}_8 \\
a_{92}x_2 + a_{93}x_3 + a_{94}x_4 + a_{95}x_5 + a_{96}x_6 + a_{97}x_7 + a_{98}x_8 + a_{99}x_9 = \tilde{b}_9 \\
a_{10,2}x_2 + a_{10,3}x_3 + a_{10,4}x_4 + a_{10,5}x_5 + a_{10,6}x_6 + a_{10,7}x_7 + a_{10,8}x_8 + a_{10,9}x_9 + a_{10,10}x_{10} = \tilde{b}_{10} \\
a_{11,2}x_2 + a_{11,3}x_3 + a_{11,4}x_4 + a_{11,5}x_5 + a_{11,6}x_6 + a_{11,7}x_7 + a_{11,8}x_8 + a_{11,9}x_9 + a_{11,10}x_{10} + a_{11,11}x_{11} = \tilde{b}_{11}
\end{cases}
$$

$$
\begin{cases}
a_{44}x_4 = b_4\\
a_{54}x_4 + a_{55}x_5 = \hat{b}_5\\
a_{64}x_4 + a_{65}x_5 + a_{66}x_6 = \hat{b}_6\\
a_{74}x_4 + a_{75}x_5 + a_{76}x_6 + a_{77}x_7 = \hat{b}_7\\
a_{84}x_4 + a_{85}x_5 + a_{86}x_6 + a_{87}x_7 + a_{88}x_8 = \hat{b}_8\\
a_{94}x_4 + a_{95}x_5 + a_{96}x_6 + a_{97}x_7 + a_{98}x_8 + a_{99}x_9 = \hat{b}_9\\
+a_{10,4}x_4 + a_{10,5}x_5 + a_{10,6}x_6 + a_{10,7}x_7 + a_{10,8}x_8 + a_{10,9}x_9 + a_{10,10}x_{10} = \hat{b}_{10}\\
+a_{11,4}x_4 + a_{11,5}x_5 + a_{11,6}x_6 + a_{11,7}x_7 + a_{11,8}x_8 + a_{11,9}x_9 + a_{11,10}x_{10} + a_{11,11}x_{11} = \hat{b}_{11}
\end{cases}
$$

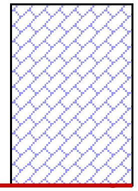
Si ottiene il seguente Sistema “Attivo”

Parte attiva del sistema triangolare

$$
\begin{cases}
a_{44}x_4 & = \hat{b}_4 \\
a_{54}x_4 + a_{55}x_5 & = \hat{b}_5 \\
a_{64}x_4 + a_{65}x_5 + a_{66}x_6 & = \hat{b}_6 \\
a_{74}x_4 + a_{75}x_5 + a_{76}x_6 + a_{77}x_7 & = \hat{b}_7 \\
a_{84}x_4 + a_{85}x_5 + a_{86}x_6 + a_{87}x_7 + a_{88}x_8 & = \hat{b}_8 \\
a_{94}x_4 + a_{95}x_5 + a_{96}x_6 + a_{97}x_7 + a_{98}x_8 + a_{99}x_9 & = \hat{b}_9 \\
a_{10,4}x_4 + a_{10,5}x_5 + a_{10,6}x_6 + a_{10,7}x_7 + a_{10,8}x_8 + a_{10,9}x_9 + a_{10,10}x_{10} & = \hat{b}_{10} \\
a_{11,4}x_4 + a_{11,5}x_5 + a_{11,6}x_6 + a_{11,7}x_7 + a_{11,8}x_8 + a_{11,9}x_9 + a_{11,10}x_{10} + a_{11,11}x_{11} & = \hat{b}_{11}
\end{cases}
$$



$$
\begin{aligned}
&a_{66}x_6 &= \ddot{b}_6\\
&a_{76}x_6 + a_{77}x_7 &= \ddot{b}_7\\
&a_{86}x_6 + a_{87}x_7 + a_{88}x_8 &= \ddot{b}_8\\
&a_{96}x_6 + a_{97}x_7 + a_{98}x_8 + a_{99}x_9 &= \ddot{b}_9\\
&+a_{10,6}x_6 + a_{10,7}x_7 + a_{10,8}x_8 + a_{10,9}x_9 + a_{10,10}x_{10} &= \ddot{b}_{10}\\
&+a_{11,6}x_6 + a_{11,7}x_7 + a_{11,8}x_8 + a_{11,9}x_9 + a_{11,10}x_{10} + a_{11,11}x_{11} &= \ddot{b}_{11}
\end{aligned}
$$

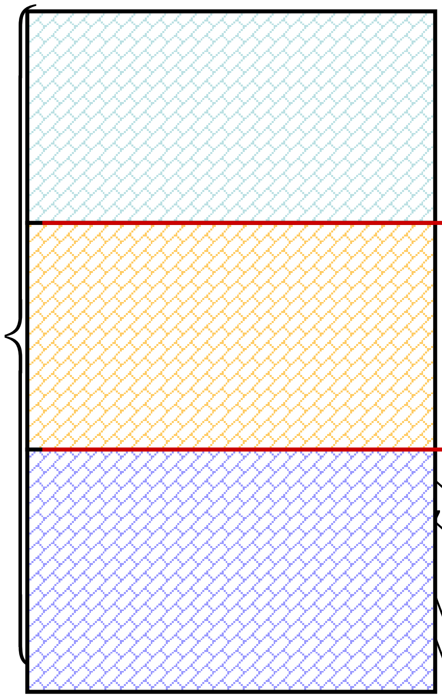


Si ottiene il seguente Sistema “Attivo”

Passo 4

Parte attiva del sistema triangolare

$$
\begin{cases}
a_{66}x_6 & = \ddot{b}_6 \\
a_{76}x_6 + a_{77}x_7 & = \ddot{b}_7 \\
a_{86}x_6 + a_{87}x_7 + a_{88}x_8 & = \ddot{b}_8 \\
a_{96}x_6 + a_{97}x_7 + a_{98}x_8 + a_{99}x_9 & = \ddot{b}_9 \\
a_{10,6}x_6 + a_{10,7}x_7 + a_{10,8}x_8 + a_{10,9}x_9 + a_{10,10}x_{10} & = \ddot{b}_{10} \\
a_{11,6}x_6 + a_{11,7}x_7 + a_{11,8}x_8 + a_{11,9}x_9 + a_{11,10}x_{10} + a_{11,11}x_{11} & = \ddot{b}_{11}
\end{cases}
$$

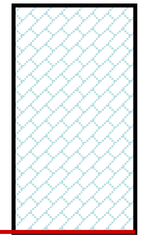


$$a_{88}x_8 = \bar{b}_8$$

$$a_{98}x_8 + a_{99}x_9 = \bar{b}_9$$

$$+a_{10,8}x_8 + a_{10,9}x_9 + a_{10,10}x_{10} = \bar{b}_{10}$$

$$+a_{11,8}x_8 + a_{11,9}x_9 + a_{11,10}x_{10} + a_{11,11}x_{11} = \bar{b}_{11}$$



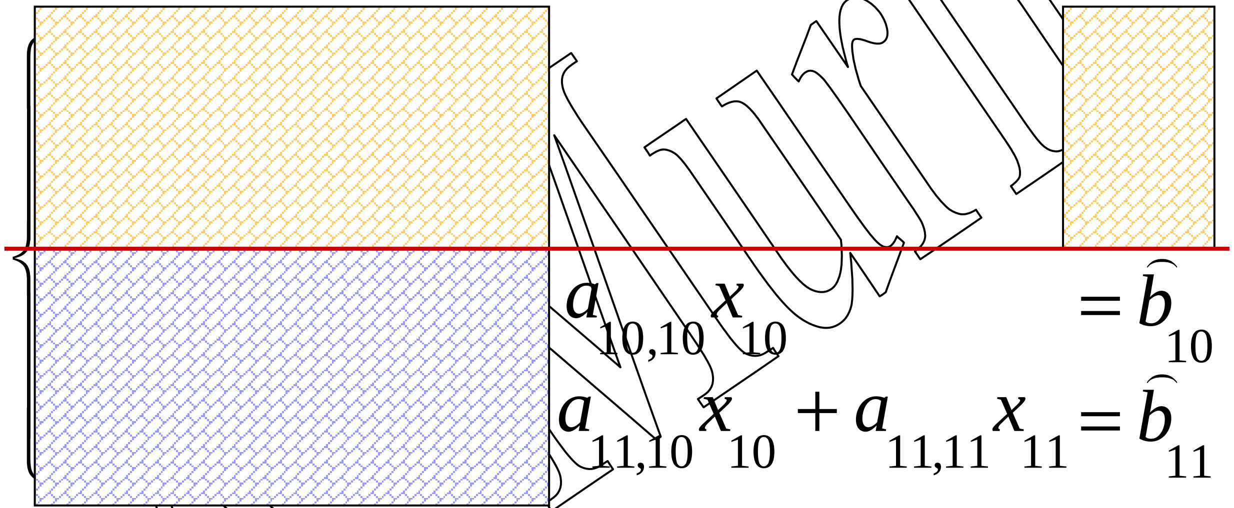
Si ottiene il seguente Sistema "Attivo"

Parte attiva del sistema triangolare

$$
\begin{cases}
a_{88}x_8 & = \bar{b}_8 \\
a_{98}x_8 + a_{99}x_9 & = \bar{b}_9 \\
a_{10,8}x_8 + a_{10,9}x_9 + a_{10,10}x_{10} & = \bar{b}_{10} \\
a_{11,8}x_8 + a_{11,9}x_9 + a_{11,10}x_{10} + a_{11,11}x_{11} & = \bar{b}_{11}
\end{cases}
$$

risolve il sistema $\mathbf{A_{44}X_4=B_4}$

Dopo il passo 5…



$$\begin{cases} a_{10,10}\,x_{10} = \hat{b}_{10} \\ a_{11,10}\,x_{10} + a_{11,11}\,x_{11} = \hat{b}_{11} \end{cases}$$

Si ottiene il seguente Sistema “Attivo”

Parte attiva del sistema triangolare

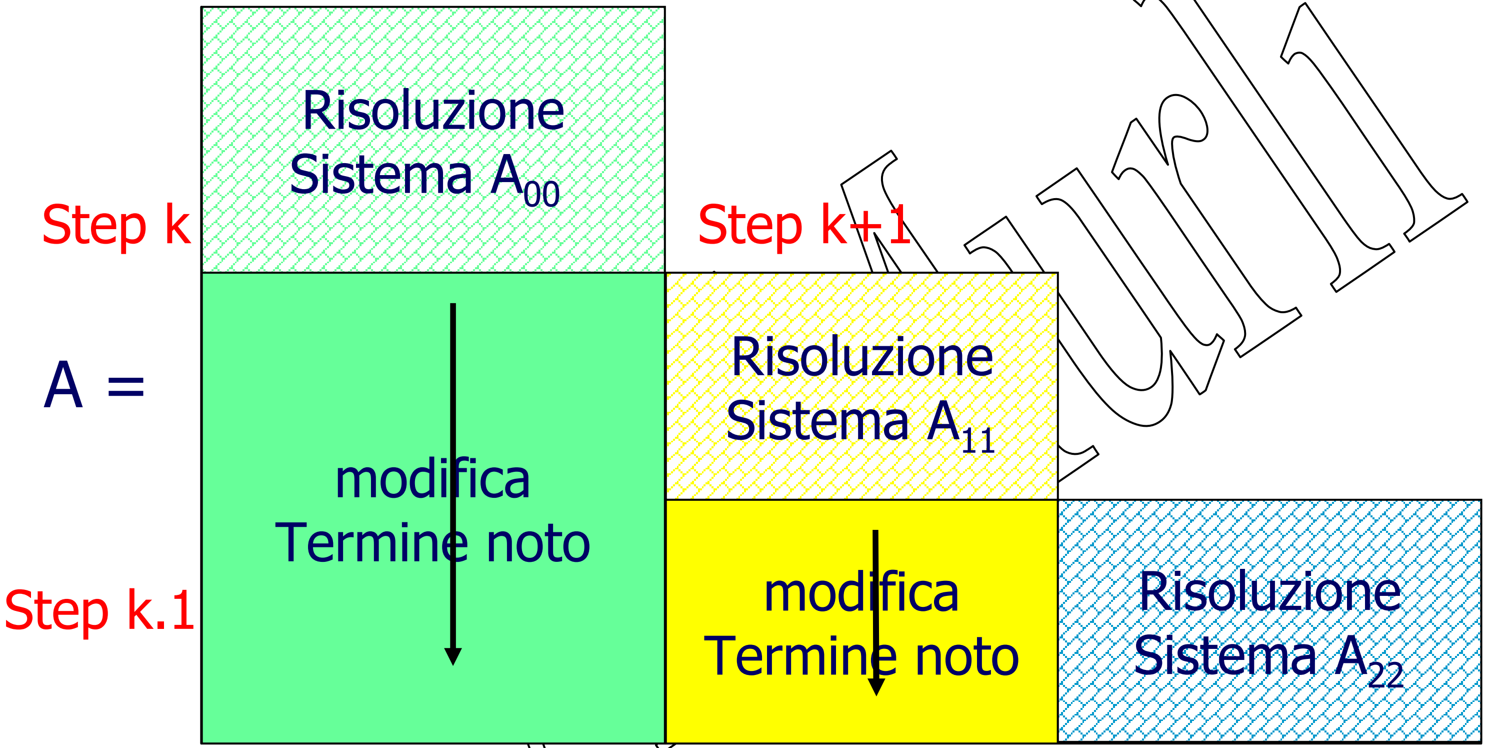
$$\begin{cases} a_{10,10}\,x_{10} = \hat{b}_{10} \\ a_{11,10}\,x_{10} + a_{11,11}\,x_{11} = \hat{b}_{11} \end{cases}$$

In generale i passi dell'algoritmo a blocchi sono

n = N/nb

Algoritmo a blocchi: Fasi di calcolo

La matrce è decomposta in 3 x 3 blocchi



L'algoritmo a blocchi procede
Per colonne

Algoritmo a blocchi

Partizioniamo la matrice , il vettore x e il vettore b in blocchi

$A_{0,0}$							X_0		B_0
$A_{1,0}$	$A_{1,1}$						X_1		B_1
$A_{2,0}$	$A_{2,1}$	$A_{2,2}$					X_2		B_2
$A_{3,0}$	$A_{3,1}$	$A_{3,2}$	$A_{3,3}$				X_3	=	B_3
$A_{4,0}$	$A_{4,1}$	$A_{4,2}$	$A_{4,3}$	$A_{4,4}$			X_4		B_4
$A_{5,0}$	$A_{5,1}$	$A_{5,2}$	$A_{5,3}$	$A_{5,4}$	$A_{5,5}$		X_5		B_5

Le matrici A_{ii} sono matrici triangolari inferiori

Algoritmo a blocchi

$$\begin{cases} A_{00}x_0 & = B_0 \\ A_{10}x_0 + A_{11}x_1 & = B_1 \\ A_{20}x_0 + A_{21}x_1 + A_{22}x_2 & = B_2 \\ \vdots \quad \vdots \quad \vdots \quad \ddots & = \vdots \\ A_{n0}x_0 + A_{n1}x_1 + A_{n2}x_2 + \cdots + A_{nn}x_n & = B_n \end{cases}$$

Passo 0: $A_{00} X_0 = B_0$

Passo 1: $B_1 = A_{10} X_0 + A_{11} X_1$

Passo 2: $B_2 = A_{20} X_0 + A_{21} X_1 + A_{22} X_2$

Passo 3: $B_3 = A_{30} X_0 + A_{31} X_1 \; A_{32} X_2 + A_{33} X_3$

Passo n: $B_n = A_{n0} X_0 + A_{n1} X_1 \; A_{32} X_2$ .... $+ A_{nn} X_n$

Algoritmo a blocchi: sequenza temporale …

0 1 2

$$\begin{cases} A_{00}x_0 & = B_0 \\ A_{10}x_0 + A_{11}x_1 & = B_1 \\ A_{20}x_0 + A_{21}x_1 + A_{22}x_2 & = B_2 \\ \vdots \quad \vdots \quad \vdots & \vdots \\ A_{n0}x_0 + A_{n1}x_1 + A_{n2}x_2 + \cdots + A_{nn}x_n & = B_n \end{cases}$$

0.1 1.1 2.1

Step k: Risoluzione sistema triangolare $A_{kk} X_k = B_k$

Step k.1: Modifica termine noto del sistema relativo alla matrice A_{k+1k+1}

Step k+1:Risoluzione sist.triangolare $A_{k+1k+1} X_{k+1} = B_{k+1}$

Fine lezione