

Evolutionary problems

Fall 2013 at Institut Mittag-Leffler

The theory of non-linear partial differential equations (PDEs) is of fundamental importance in mathematical analysis and through recent breakthroughs and insights it has reached a stage where some difficult and important questions can be fruitfully addressed.

It is also one of the cornerstones of the Nordic School of Mathematical Analysis. In order to understand the dynamics, it is essential to consider evolutionary problems as well. Indeed, there are several phenomena and difficulties which are not present in the time independent case. For example, disturbances may have a finite speed of propagation or there may be extinction in finite time. These are often natural properties in applications and challenging mathematical problems appear.

This one semester program at Institut Mittag-Leffler will focus on a large class of singular and degenerate PDEs ranging from flows by mean curvature to the infinity Laplacian. Although these equations have a common structure, they are connected to many different applications such as the diffusion in highly non-homogeneous and anisotropic media, the motion of multi-phase fluids and flow through porous media. The velocity gradient of these fluids depends in a non-linear way on the stress tensor.

Other applications include behaviour of composite materials, image processing, stochastic game theory, and pricing the assets in financial markets.

A crucial role in understanding non-linear phenomena is played by regularity estimates based only on the structure of the equations. Indeed, solutions of non-linear PDEs, when they exist, are not a priori smooth. During the last decades a vast amount of research papers have been published and powerful techniques have been developed to understand the behaviour of weak solutions.

In several ways the recent advances open up a whole new area of research similar to the progress that started a few decades ago concerning regularity and free boundary problems for linear PDEs. These new methods have already turned out to be powerful enough to solve previously unreachable problems.

Despite some progress, the overall picture remains unclear and open problems abound. In this program we consider four themes which have been gaining traction recently. As will become apparent, the novelties are not mutually exclusive and several of the themes can overlap in one research problem. The themes are:

1. boundary behavior,
2. theory for systems,
3. free and moving boundaries, and
4. connections to stochastic game theory.

The list is not exhaustive. For example, the extensions to other degenerate structures are important issues, and so far there are only partial results available. This pertains for instance to equations of porous medium type, doubly nonlinear equations, the thin film or the quantum drift diffusion equations, and the use of so-called rough paths to describe dynamics.

Finally, a coupling of degenerate structures, such as doubly degenerate structures, is of great importance since many equations and systems arising in applications possess such structures.

A collaborative effort of a diverse group of researchers in PDEs, which jointly masters many of the tools of the trade, will be needed to attack these research problems. Thus we have every reason to believe that results in many of the problems will be obtained during a semester at Institut Mittag-Leffler.

Organizing Committee:

Frank Duzaar, University of Erlangen

Juha Kinnunen, (contact person) Aalto University

Jan Kristensen, Oxford University

Peter Lindqvist, Norwegian University of Science and Technology

Giuseppe Mingione, University of Parma

Kaj Nyström, University of Umeå