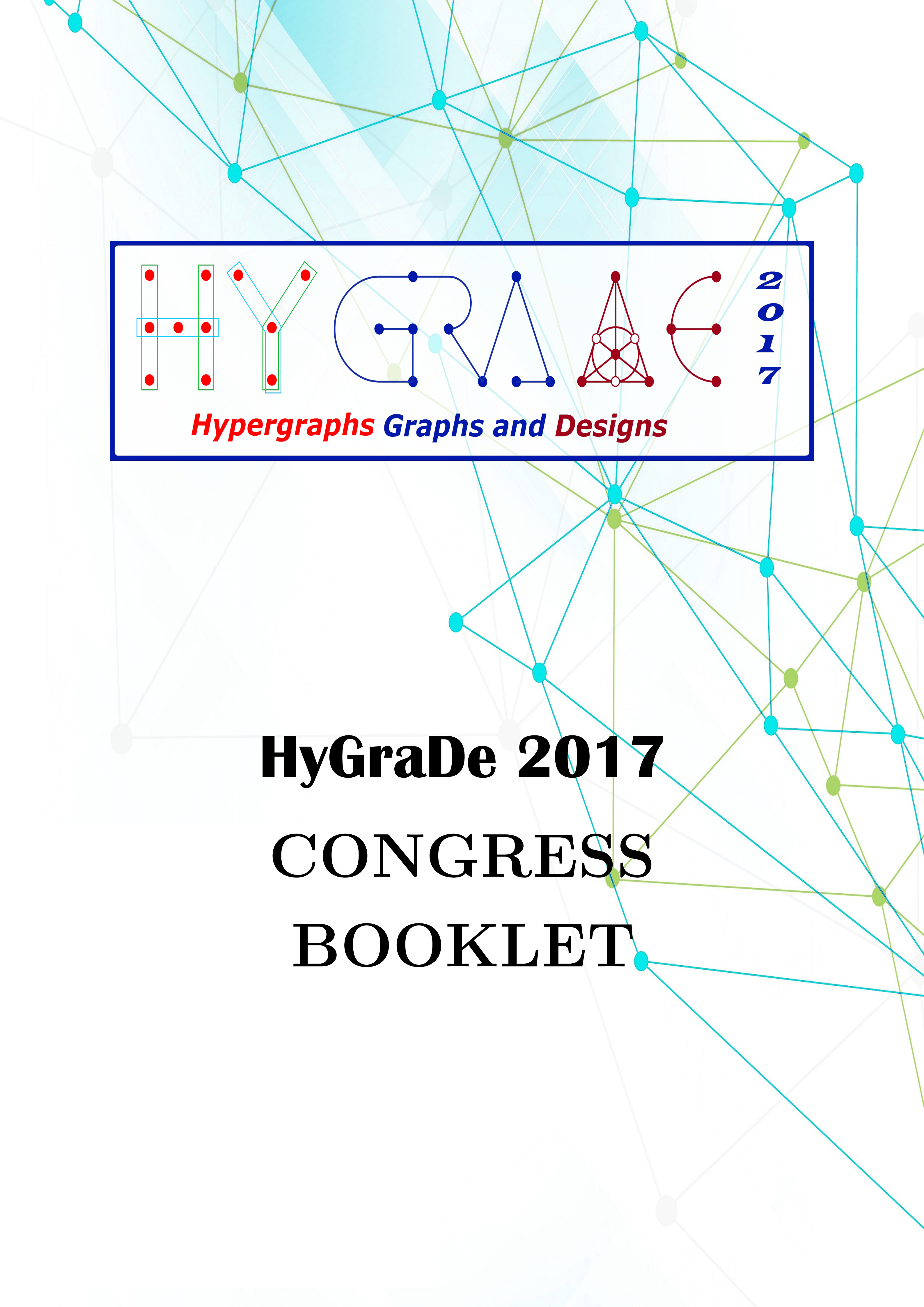

HYGRADE 2017

Hypergraphs Graphs and Designs

HyGraDe 2017

CONGRESS BOOKLET

Hypergraphs, Graphs and Designs
HyGraDe 2017

Celebrating Mario Gionfriddo's 70th Birthday

Sant'Alessio Siculo (ME), Italy, 20th June – 24th June 2017

Scientific Committee
M. Buratti, G. Lo Faro, G. Lunardon, M. Milanič

Organising Committee
F. Belardo, L. Giuzzi, E.M. Li Marzi, G. Lo Faro, L. Milazzo, S. Milici, A. Tripodi

Supporting Organizations

Università di Catania

Università di Messina

Università di Napoli

Accademia Peloritana

Università del Litorale

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Overview

Presentation

The International Congress "Hypergraphs, Graphs and Designs - HyGraDe 2017" takes place at Elihotel in the city of Sant'Alessio Siculo, Italy, from June 20th to June 24th, 2017. The present booklet contains the list of the participants and the abstracts of the presentations which will be given during the conference.

Topics of interest include, but are not limited to:

- Hypergraphs;
- Graphs;
- Combinatorial Designs.

The above topics have been the main interest of our Colleague Mario Gionfriddo who has dedicated his studies and introduced a number of scholars in these investigations. This year Mario is 70 years old and he will retire. Therefore we, the Organizing Committee, have decided to organize this event to show him our gratitude for all his efforts during his career. Most invited speakers are long-time friends and co-authors of Mario in many papers. Most contributed talks are delivered from people who are connected to Mario or inspired by Mario's researches.

Invited Speakers

- Richard Brualdi - University of Wisconsin (USA)
- Marco Buratti - University of Perugia (IT)
- Charles Colbourn - Arizona State University (USA)
- Klavdija Kutnar - University of Primorska (SLO)
- Josef Lauri - University of Malta (MAL)
- Charles C. Lindner - University of Auburn (USA)
- Dragan Marušič - University of Primorska (SLO)
- Alexander Rosa - McMaster University (CAN)
- Zsolt Tuza - Hungarian Academy of Sciences (HUN)
- Vitaly Voloshin - Troy University (USA)

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Congress Venue

Sant'Alessio Siculo is a city in the province of Messina facing the Ionian sea and close to the well-known city of Taormina. It had the name of Arghenrion Akron which after is named "Silver Cape" by Tolomeo Alessandrini. To the right, the city is delimited by a promontory on which stands the Saracen castle of the twelfth century. The manor has a polygonal plan topped by a cylindrical tower (unfortunately, it is closed to public). Now Sant'Alessio Siculo has become a tourist center, with its pebble beach, and thanks mainly to tourist developments and to establishments that have been realized in recent years.

A bit of History *by Mario Gionfriddo*

Graph Theory and Block-Designs Theory in Sicily have very precise dates of origin. Graph Theory in Sicily started in 1969 with some seminars given by F. Speranza, Full Professor of Differential Geometry at University of Messina. F. Speranza continued his studies in Differential Geometry and Graph Theory, later devoting his attention to Mathematics Education and Mathematical Logics.

At that time the graphs were called singrams, and that was the name used for graphs in some of Speranza's research papers. A few years later, it was Speranza who pushed me to study Graph Theory.

Block-Designs Theory in Sicily began in May 1978 with seminars hosted by C.C. Lindner at University of Messina. Furthermore, we can precisely say that the first paper in Sicily on Steiner Systems is [M. Gionfriddo, C.C. Lindner, Construction of Steiner quadruple systems having a prescribed number of blocks in common, Discrete Mathematics 34 (1981), p. 31-42]. Lindner came to Messina as Visiting Professor covered by a grant of INdAM-GNSAGA.

So, a long scientific cooperation had started which still continues today. After myself, many other researchers from Messina and Catania of Designs Theory have cooperated with him and they have established strong scientific cooperations, that are alive even today.

In 1981, I moved to Catania after being offered a Full Professorship position at the University of Catania. Therefore, we can say that 1981 was the year that Graph Theory and Design Theory were introduced to Catania. In 1986 I organized the Congress "1st International Catania Combinatorial Conference: Graphs, Steiner Systems and their applications" held in Santa Tecla, September 12-17, 1986. Many well-known scholars of graph theory and hypergraphs attended this congress, among them, C. Berge, F. Harary, C. Thomassen, S. Simić, S. Fiorini, G. Sabidussi, Zs. Tuza. As well as recognized researchers of Design Theory, such as C. Coulbourn, C.C. Lindner, A. Rosa, C. Rodger, L. Teirlink, K. Phelps.

The second congress followed three years later, as the "2nd International Catania Combinatorial Conference: Graphs, Designs and Combinatorial Geometries", Santa Tecla, September 3-9, 1989, with the notable presence of Paul Erdős, surely one of the greatest mathematicians of the XX century.

The third congress took place in 1992. On that occasion, the congress became part of the International Congress Series "Combinatorics", which has been organized in Italy every two years till today: "3rd International Catania Combinatorial Conference: Combinatorics 1992", Santa Tecla, September 12-17, 1992. In that event, the number of participants exceeded 250, including once again among others Paul Erdős. After that congress a long series of yearly workshops stepwise organized in Catania, Messina and also Malta with J. Lauri. At the same time, since 1993, V. Voloshin from University of Kinishev in Moldavia had started to visit the Departments of Mathematics in Catania and Messina: the scientific achievements obtained thanks to him are numerous, relevant and of great quality. A scientific cooperation that even today is of current relevance. Vitaly Voloshin is today Full Professor at Troy University in Alabama (USA).

In 2004, another international congress of the "Combinatorics" series took place, with the usual unavoidable success in terms of participants and communications of great level: 4th International Catania Combinatorial Conference Combinatorics 2004" Capomulini, September 12–17, 2004.

Mario Gionfriddo – a short CV

Mario Gionfriddo was born in Messina on November 6th, 1946. He completed Primary and Secondary School in Messina and thereafter he studied Mathematics attaining a Degree at the University of Messina.

From 1970 he worked as Research Fellow at the University of Messina, and in 1974 he became “Assistente Ordinario“ (Assistant Professor) at the same university. In 1981 he achieved the position of Full Professor at the University of Catania, which has remained his affiliation university until his retirement (October 2017).

During his time at the University of Catania, Mario has been the Principal Investigator of many local and national research projects in the field of Discrete Mathematics, especially for Graph Theory. He took the role of President of the Degree Course in Mathematics (1994-1996) and Dean of Mathematics Department (1996-1999).

He organized many successful workshops and congresses in Sicily, including two in the series “Combinatorics”, and the corresponding conference proceedings were covered in well-known journals, including “Ars Combinatoria”, “Discrete Mathematics” and others, where he acted as Guest Editor.

He has been a member of the editorial board of the following journals: “Le Matematiche”, “J. Inf. Opt. Sc.”, “J. Combinatorics Inf. Syst. Sc.”, “J. Discrete Mathematical Sciences”, “J. Interdisciplinary Mathematics” and “Isrn-Combinatorics”.

In his career he has written more than 140 papers on Graph Theory with around 40 co-authors, and a monograph titled “Hypergraphs and Designs” co-authored with Lorenzo Milazzo and Vitaly Voloshin.

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Invited talks

Main speakers

Alternating Sign Matrices and Hypermatrices

Richard A. Brualdi
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Alternating Sign Matrices (ASMs) are square $(0, \pm 1)$-matrices such that, ignoring 0's, the $+1$'s and -1's in each row and column alternate beginning and ending with a $+1$. Permutation matrices are the ASMs without any -1's. We shall discuss the origins and properties of ASMs. There is a partial order on permutation matrices, the so-called Bruhat order, which extends in a very natural and surprising way to ASMs. This partial order is ranked and has many interesting properties. There are hypermatrix generalizations of permutation matrices which lead to hypermatrix generalizations of ASMs and latin squares. This talk is taken from joint work with Geir Dahl and joint work with Michael Schroeder.

Designs and graph decompositions over finite fields

Marco Buratti
(joint work with Anamari Nakic)
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A 2-(v, K, λ) design over the finite field $\mathbb{F}_q$ is a collection $\mathcal{S}$ of subspaces of the vector space $\mathbb{F}_q^v$ with dimensions from the set K and the property that any 2-dimensional subspace of $\mathbb{F}_q^v$ is contained in exactly λ members of $\mathcal{S}$. Of course it can be also viewed as a collection $\overline{\mathcal{S}}$ of subspaces of the projective space $\mathrm{PG}(v-1,q)$ with dimensions from $\{k-1 \mid k \in K\}$ such that any two distinct points belong to exactly λ members of $\overline{\mathcal{S}}$. It is trivial when $K = \{2\}$. When $K = \{k\}$ is a singleton, one simply writes "k" rather than "$\{k\}$".

To construct these designs seems to be quite hard. Indeed, in spite of the fact that the topic received a considerable amount of attention over the years, the well-celebrated 2-$(13, 3, 1)$ design over $\mathbb{F}_2$ recently obtained by Braun et al. [1] with the use of the Kramer-Mesner method is the only non-trivial example having $\lambda = 1$ known at this moment. Also, for $\lambda > 1$ only few theoretical constructions are known. Among them, we have the existence of a 2-$(v, 3, 7)$ design over $\mathbb{F}_2$ for any $v \equiv \pm 1 \pmod 6$ obtained by Thomas [2].

In the first part of my talk I will show how we used difference methods in order to extend Thomas result to the case $v \equiv 3 \pmod 6$ and to multiple dimension sizes.

In the second part, starting from the very well-known remark that a classic 2-(v, k, λ) design can be viewed as a decomposition of the λ-fold of the complete graph of order v into cliques of size k, I will propose the new notion of a *graph decomposition over a finite field* presenting several concrete constructions such as a decomposition of the complete graph on the points of $\mathrm{PG}(6, 2)$ into heptagons each of which has vertex-set coinciding with the point-set of a plane.

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[2] S. Thomas, Designs over finite fields, *Geom. Dedicata*, 93 (1987), 237–242.

Asymptotic Sizes of Covering Arrays

Charles J. Colbourn
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Covering arrays are used to test the correctness of complex engineered systems with k components each having v options, when collections of at most t component options can cause failures. Of most interest are cases when $2 \leq t \leq 6$ and $2 \leq v \leq 10$, but k can be quite large, perhaps in the hundreds or thousands. For this reason, asymptotic existence results bounding the sizes of covering arrays as a function of the number of components have been of interest. For decades, the only real improvement on the simple probabilistic argument used the Lovász Local Lemma. Moreover, these probabilistic arguments had limited impact on the explicit construction of covering arrays for practical use.

Recently, many improvements on the asymptotic bounds have been obtained by Godbole, Francetic and Stevens, and Sarkar and the presenter. The methods to obtain these employ varying the sampling strategy, reducing the sample space, and oversampling with postprocessing. We outline these methods and discuss their asymptotic consequences. Perhaps surprisingly, we show that each method leads to an efficient algorithm for constructing covering arrays that not only meet or improve upon the corresponding bound, but also improve upon the best covering arrays previously constructed.

Vertex-transitive-odd numbers

Klavdija Kutnar
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Following [*Ars Math. Contemp.* **10** (2016), 427–437], an automorphism of a graph is said to be *even/odd* if it acts on the vertex set of the graph as an even/odd permutation. A positive integer n is said to be a *vertex-transitive-odd number* (in short, a *VTO-number*) if every vertex-transitive graph of order n admits an odd automorphism. In this talk I will present recent results on this topic: There exists infinitely many VTO numbers which are square-free and have arbitrarily long prime factorizations. Cayley numbers congruent to 2 modulo 4, cubefree nilpotent Cayley numbers congruent to 3 modulo 4, and numbers of the form $2p$, p a prime, are VTO numbers. At the other extreme, for a positive integer n the complete graph K_n and its complement are the only vertex-transitive graphs of order n admitting odd automorphisms if and only if n is a Fermat prime.

This is a joint work with Ademir Hujdurović and Dragan Marušič.

Colouring σ-hypergraphs

Josef Lauri
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I find that one of the most intriguing aspects in the colouring of mixed hypergraphs is that their chromatic spectrum can be broken, that is, the hypergraph can be coloured using k_1 or k_3 colours but not with k_2 colours, where $k_1 < k_2 < k_3$. This research arose out of the attempt to try to investigate and explain this phenomenon. I shall my attention to uniform bi-hypergraphs, in which the constraint is that the set of vertices making up every hyperedge must be given at least two colours but must also have at least one repeated colour. I shall describe early attempts to grapple with this problem starting from some ideas I investigated with Lucia Gionfriddo, then to the first paper with Yair Caro in which we introduced the notion of σ-hypergraphs, followed up with work with our then PhD student Christina Zarb.

The definition of a σ-hypergraph is quite simple. We envisage that the vertices are arranged in a $q \times n$ array and that the integer r and a partition σ of r are given. Then the hyperedges are defined as all r-subsets E of vertices such that the sizes of the nonempty intersections of E with the columns of the array form the partition σ. By manipulating the parameters n, q, r and σ we can force the σ-hypergraph to have a broken or a continuous chromatic spectrum. This gives us a handle with which we can investigate the phenomenon of broken spectra, and it is this investigation, done together with Yair Caro and Christina Zarb, which I shall focus mainly on.

Almost 2-Perfect 6-Cycle Systems

Charles C. Lindner
(joint work with Alex Rosa and Mariusz Meszka)
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A 6-cycle system (X, C) is said to be almost 2-perfect if it is possible to place a 6-cycle inside each 6-cycle in C so that the resulting collection of 6-cycles is a 6-cycle system. Not too surprisingly sometimes this is possible and sometimes its not. For example the 6-cycle system (X, C_1) is almost 2-perfect.

$$X = \{0,1,2,3,4,5,6,7,8\}$$

$$C_1 = \begin{cases} (0,1,2,3,4,5) & \to (0,2,4,1,5,3) \\ (0,2,4,1,6,7) & \to (0,4,6,2,7,1) \\ (0,3,7,8,4,6) & \to (0,7,4,3,6,8) \\ (0,8,6,5,7,4) & \to (0,6,7,8,4,5) \\ (1,7,2,8,5,3) & \to (1,2,5,7,3,8) \\ (1,8,3,6,2,5) & \to (1,3,2,8,5,6) \end{cases}$$

The 6-cycle system (X, C_2) is *not*!

$$X = \{0,1,2,3,4,5,6,7,8\}$$

$$C_2 = \begin{cases} (0,1,2,3,4,5) \\ (0,2,4,1,3,6) \\ (0,3,5,1,7,8) \\ (0,4,6,8,2,7) \\ (1,6,5,7,3,8) \\ (2,5,8,4,7,6) \end{cases}$$

This is an elementary survey showing that the spectrum for almost 2-perfect 6-cycle systems is the set of all $n \equiv 1$ or $9 \pmod{12}$, (= the spectrum for 6-cycle systems). This can be extended to maximum packings.

Symmetric graphs: why semiregularity matters

Dragan Marušič
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In this talk I will discuss the still open problem of existence of semiregular automorphisms in vertex-transitive (di)graphs (that is, a non-identity automorphism with all orbits of the same size) and its generalization to 2-closed groups. In particular, I will focus on the importance of the semiregular automorphism to various other open problems in graph theory.

Reaction graphs of combinatorial configurations

Alexander Rosa
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The concept of reaction graphs has its origins in chemistry. The reaction graphs of interest to us correspond to describing small changes or rearrangements of labelled configurations, such as graphs, designs, graph decompositions and similar.

We illustrate this concept on several examples some of which lead to interesting graphs.

Mixed hypergraphs and beyond

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The concept of *mixed hypergraph* coloring was introduced by Voloshin in 1993. Mixed hypergraphs are composed of so-called C-edges and D-edges, with the requirement that every feasible coloring assigns a common color to at least two vertices of each C-edge, and distinct colors to at least two vertices of each D-edge.

Generalizing this structure class, a decade ago Bujtás and Tuza introduced *stably bounded hypergraphs*, each edge E of which may have four restrictions s, t, a, b with the following meaning:

- E contains vertices of at least s distinct colors;
- E contains vertices of at most t distinct colors;
- some color occurs in E at least a times;
- each color occurs in E at most b times.

Hence, inside each edge E, the values s and t bound the number of colors, while a and b bound the maximum multiplicity of colors. In the general setting, the values s, t, a, b may be non-uniform over the edges, i.e. they are four functions from the edge set to the set of positive integers. The theory of these kinds of coloring is quite developed, but still there are several challenging problems and conjectures which remained open since many years.

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Mario Gionfriddo and Mixed Hypergraph Coloring

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We plan to describe the origin and development of Mixed Hypergraph Coloring from its inception in 1993. Especially we will discuss the implicit and explicit contribution of Professor Mario Gionfriddo to this theory, its applications to Coloring of Block Designs and his outstanding role in Italian and especially Sicilian Discrete Mathematics, namely, the Theory of Graphs, Hypergraphs and Designs.

Contributed talks

List of contributors

Treelike Snarks

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We study snarks whose edges cannot be covered by fewer than five perfect matchings. Esperet and Mazzuoccolo found an infinite family of such snarks, generalising an example provided by Hägglund. We construct another infinite family, arising from a generalisation in a different direction. The proof that this family has the requested property is computer-assisted. In addition, we prove that the snarks from this family (we call them *treelike snarks*) have circular flow number $\phi_C(G) \geq 5$ and admit a 5–cycle double cover.

Equitable colourings for systems of 4-kites

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An *equitable colouring* of a balanced G-design $(X, \mathcal{B})$ is a map $\phi \colon \mathcal{B} \to C$ such that $|b_i(x) - b_j(x)| \leq 1$ for any $x \in X$ and i, j, with $i \neq j$, being $b_i(x)$ the number of blocks containing the vertex x and coloured with the colour i. A c-colouring is a colouring in which exactly c colours are used. A c-colouring of type s is a colourings in which, for every vertex x, all the blocks containing x are coloured exactly with s colours. A bicolouring, tricolouring or quadricolouring is an equitable colouring with $s = 2$, $s = 3$ or $s = 4$. We consider systems of graphs consisting of a 4-cycle and a pendant edge. We call such a graph a 4-kite and we consider balanced 4-kite systems. In particular, it can be proved that c-bicolourings of balanced 4-kite systems exist if and only if $c = 2, 3$.

Multigraphs with relatively large palette-index

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For a given (proper) edge-coloring of a graph G the palette of a vertex x is simply the set of colors received by the edges which are incident with x. The palette index $\check{s}(G)$ is the minimum number of palettes occurring among all edge-colorings of G.

This definition was given in [1] for the case in which G is a simple undirected graph, but it can clearly be extended *verbatim* to multigraphs which are loopless and undirected.

Meaningful upper bounds for the palette index $\check{s}(G)$ in terms of some parameter of the graph can be obtained in some circumstances. For instance, if G is a class 2 regular graph of degree Δ, then it is not hard to prove that the inequality $\check{s}(G) \leq \Delta + 1$ holds. As far as we know, the unique general upper bound for the palette index of a simple graph G in terms of its maximum degree Δ is $\check{s}(G) \leq 2^{\Delta+1} - 2$, stating that the number of palettes in a $(\Delta + 1)$-edge-coloring of G cannot exceed the total number of admissible subsets of the color-set.

On the other hand, it is immediately seen that the palette index of a star with maximum degree Δ grows linearly in Δ, while a family of trees with maximum degree Δ whose palette index asymptotically behaves as $\Delta \ln(\Delta)$ was presented in [2].

In this talk I want to outline the construction of a multigraph with maximum degree Δ whose palette index can be expressed quadratically in terms of Δ. The possibility of obtaining the same result without multiple edges is under consideration.

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Signed bicyclic graphs with extremal least Laplacian eigenvalue

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A signed graph is a pair $\Gamma = (G, \sigma)$, where $G = (V(G), E(G))$ is a graph and $\sigma : E(G) \to \{+1, -1\}$ is the sign function on the edges of G. For a signed graph we consider the Laplacian matrix defined as $L(\Gamma) = D(G) - A(\Gamma)$, where $D(G)$ is the matrix of vertices degrees of G and $A(\Gamma)$ is the (signed) adjacency matrix. The least Laplacian eigenvalue is zero if and only if the signed graph is balanced, i.e. all cycles contain an even number of negative edges.

Among the unbalanced bicyclic signed graphs of given order $n \geq 5$, it turns out that the least Laplacian eigenvalue is minimal for two triangles, only one of which is unbalanced, connected by a path. Such graph minimizes the least eigenvalue even in the larger set $\mathcal{N}(n)$ of not-necessarily connected graphs whose Laplacian eigenvalues are all positive and $|E(G)| = |V(G)| + 1$.

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On the length of the total domination game

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The total version of the domination game was introduced by Henning, Klavžar, and Rall [1]. This is a two-person competitive optimization game, where the players, Dominator and Staller, alternately select vertices of an isolate-free graph G. Each vertex chosen must strictly increase the number of vertices totally dominated. This process eventually produces a total dominating set D of G. Dominator wishes to minimize the number of vertices chosen in the game, while Staller wishes to maximize it. The game total domination number of G, $\gamma_{\rm tg}(G)$, is the number of vertices chosen when Dominator starts the game and both players play optimally.

A general bound on the game total domination number was established in [2] where it is shown that if G is a graph on n vertices in which every component contains at least three vertices, then $\gamma_{\rm tg}(G) \le \frac{4}{5}n$. In the same paper [2], the authors posted the conjecture which states that the sharp upper bound is $\frac{3}{4}n$. Here, we take a step forward and prove that $\gamma_{\rm tg}(G) \le \frac{11}{14}n$ holds for every G which contains no isolated vertices or isolated edges. In the proof, the total domination game is modeled as a transversal game on the open neighborhood hypergraph.

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Simple Heffter arrays and orthogonal cyclic cycle systems

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The concept of Heffter array has been introduced by A. Archdeacon in [1] where he showed various of its applications. This leads several authors to investigate the existence problem (see, for example, [2] and [4]).

Here we are interested in the relationship between Heffter arrays and orthogonal cyclic cycle systems. In this regard we introduce, in [3], the class of globally simple Heffter arrays whose existence assures the one of orthogonal cyclic k-cycle decompositions of complete graphs and of cocktail party graphs.

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Maximum scattered subspaces and maximum rank distance codes

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Let V be an r-dimensional vector space over $\mathbb{F}_{q^n}$, $n, r > 1$, and let U be an $\mathbb{F}_q$-subspace of V.If the one-dimensional $\mathbb{F}_{q^n}$-spaces of V meet U in $\mathbb{F}_q$-subspaces of dimension at most one, then U is called *scattered* (w.r.t. the Desarguesian spread).In [1] Blokhuis and Lavrauw proved that the rank of a scattered subspace is at most $rn/2$, they also showed that this bound can always be achieved when n is even. Later, existence results and explicit constructions were given for infinitely many values of r, n, q (rn even) but there were still infinitely many open cases. In this talk I will present examples of scattered subspaces with rank $rn/2$ for every values of r, n, q (rn even) [2]. Scattered subspaces of this rank will be called maximum scattered. An $\mathbb{F}_q$-linear maximum rank distance code (or MRD-code) $\mathcal{M}$ with parameters $(m, n, q; d)$ is an $\mathbb{F}_q$-subspace of the vector space of $m \times n$ matrices over $\mathbb{F}_q$ such that the non-zero matrices of $\mathcal{M}$ have rank at least d and the size of $\mathcal{M}$ reaches the theoretical upper bound $q^{\max\{m,n\}(\min\{m,n\}-d+1)}$. In [4] Sheekey showed that maximum scattered $\mathbb{F}_q$-subspaces of a 2-dimensional $\mathbb{F}_{q^n}$-space yield MRD-codes with parameters $(n, n, q; n-1)$. I will present some recent results regarding MRD-codes arising from maximum scattered subspaces. In particular, a generalization of Sheekey's result [2] and new families of MRD-codes [3].

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The spectrum of $P^{(h)}(h-1,h+1)$-designs

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Given an hypergraph $H^{(h)}$, uniform of rank h, an $H^{(h)}$-*design* [or also a design of type $H^{(h)}$] of order v is a pair $\Sigma = (X, \mathcal{B})$, where X is a set of cardinality v and $\mathcal{B}$ is a collection of hypergraphs, all isomorphic to $H^{(h)}$, such that every h-subset of X is an edge in exactly one hypergraph $H^{(h)} \in \mathcal{B}$. An *hyperpath* $P^{(h)}(k, 2h-k)$ is an hypergraph uniform of rank h, having two edges with exactly k vertices in common. It is well known the spectrum $P^{(h)}(h-1,h+1)$-designs for $h = 2, 3$. The authors determine the spectrum of $P^{(h)}(h-1,h+1)$-designs for every $h \geq 4$. Further, for $h = 4$, they determine the spectrum in all the possible cases in which $H^{(h)}$ is an hyperpath with two edges: exactly for $P^{(4)}(3,5)$-designs, $P^{(4)}(2,6)$-designs, $P^{(4)}(1,7)$-designs.

Catalan Hypercubes

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There are $n!$ paths going from 0^n to 1^n in the oriented hypercube Q_n defined over the set of n-binary words. Each path p in Q_n encodes a permutation π_p of size n, where π_p has entry i placed in position j, if the i-th step of p creates an entry 1 in position j. For a fixed permutation pattern σ of length three, the *Catalan* hypercube $Q_n(\sigma)$ [2] is obtained by removing edges from Q_n in such a way $p \to \pi_p$ maps bijectively paths from 0^n to 1^n in $Q_n(\sigma)$ onto permutations of size n that do not contain σ as a pattern. The number of permutations of size n avoiding σ—and thus the number of paths in $Q_n(\sigma)$—is the n-th Catalan number

$$c_n = \frac{1}{n+1}\binom{2n}{n},$$

from which the term "Catalan" hypercube.

Catalan hypercubes provide a natural setting for studying the graph structure of permutations avoiding patterns of length three. We show several order theoretic and combinatorial properties of $Q_n(\sigma)$, focusing on the two non-isomorphic cases $Q_n(123)$ and $Q_n(132)$. First, we study the number of (strict) intervals e_n and the number $e_{n,\ell}$ of intervals of given length ℓ in the poset $\big(Q_n(\sigma), \preceq\big)$, where $\preceq$ is the order induced by the orientation of the edges in $Q_n(\sigma)$. For both $\sigma = 123, 132$ we find

$$e_n = 2^{n-3}(n^2 + 3n) \quad \text{and} \quad e_{n,\ell} = (\ell + 1)2^n + \mathcal{O}(n^\ell).$$

Interestingly, from the latter equality it follows that asymptotically a random node of $Q_n(\sigma)$ has on average $\ell + 1$ nodes above it at distance ℓ—a quantity that does not depend on n. Second, we derive explicit formulas for the number $T_\sigma(w)$ of paths in $Q_n(\sigma)$ intersecting a given node w:

$$\begin{aligned} T_{123}(w) &= \binom{Z+z}{Z} \cdot \frac{Z-z+1}{Z+1} \cdot \binom{U+u}{U} \cdot \frac{U-u+1}{U+1}, \\ T_{132}(w) &= \binom{Z+z}{Z} \cdot \frac{Z-z+1}{Z+1} \cdot \prod_{i=1}^{b} c_{u_i}, \end{aligned}$$

where Z (resp. U) is the number of 0's (resp. 1's) in w, z (resp. u) is the length of the maximal prefix (resp. suffix) of w containing only 0's (resp. 1's), b is the number of blocks of consecutive 1's in w, and u_i is the length of the i-th block of 1's. Third, given a path p of Q_n, we ask for the number of paths of $Q_n(\sigma)$ that intersect p only in 0^n and 1^n. We solve some instances of this problem, deriving as a corollary the number of indecomposable permutations [1] of size n avoiding σ. Finally, we discuss possible applications of Catalan hypercubes $Q_n(\sigma)$ in modeling accessibility phenomena in random fitness landscapes [3]: paths through the nodes of the hypercube represent the possible evolutionary histories of a gene that can be affected by binary mutations $0 \to 1$ at its n positions (loci). Each node of the hypercube has a random fitness value, and the probability of a fitness increasing path from a global minimum 0^n to a global maximum 1^n is investigated.

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A zero-sum problem in the theory of block designs

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We have shown that symmetric and affine 2-designs $D = (P, B)$ can be embedded in a finite commutative group in such a way that the blocks are exactly the k-subsets of elements in P which sum up to zero, whereas the only Steiner triple systems which have this property are the point-line designs of $AG(d, 3)$ and $PG(d, 2)$.

This leads to the following question. Let B_k be the family of k-subsets summing to zero in an elementary abelian group P. Is it possible to settle whether $D = (P, B_k)$ is a 2-design and find its parameters?

By introducing a (possibly) new technique, we answer affirmatively the question.

Decompositions of the complete n-partite equipartite multigraph with any minimum leave and minimum excess

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A decomposition of $\lambda K_{n(g)} \setminus L$, the complete n-partite equipartite multigraph with a subgraph L (called the leave) removed, into edge disjoint copies of a graph G is called a maximum group divisible packing of $\lambda K_{n(g)}$ with G if L contains as few edges as possible. A decomposition of $\lambda K_{n(g)} \cup E$, the complete n-partite equipartite multigraph union a graph E (called the excess), into edge disjoint copies of a graph G is called a minimum group divisible covering of $\lambda K_{n(g)}$ with G if E contains as few edges as possible.

We continue Billington and Lindner's work in [1] to examine all possible minimum leaves for maximum group divisible packings of $\lambda K_{n(g)}$ with G and all possible excesses for minimum group divisible coverings of $\lambda K_{n(g)}$ with G, where G is a triangle K_3, or a triangle plus one dangling edge $K_3 + e$, or $K_4 - e$ [2, 3]. When G is K_4, the problem is closely related with many other combinatorial configurations, such as balanced sampling plans excluding contiguous units, matching divisible designs, etc. We shall show that the obvious divisibility conditions are sufficient for the existence of matching divisible designs with block size four [4].

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Configurations in Sicily before 1910 and after 1986

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My talk will discuss the research on configurations in Sicily before 1910 (historical part), followed by the investigation of configurations after 1986 which was started in the Acireale conference of 1986.

Configurations are linear regular uniform hypergraphs, mainly discussed in a geometrical language, and closely related to bipartite graphs, combinatorial designs and similar structures. A small but already quite interesting example is the Fano configuration with 7 points and 7 lines, corresponding to the Heawood graph. In order to exist the parameters of a configuration have to fulfill certain necessary conditions. In general, it has to be investigated whether these conditions are also sufficient, and if yes, how many non-isomorphic structures there are and what properties they have.

It was Vittorio Martinetti in Messina who in 1886 published his paper on configurations and started the ”Sicilian research”. He later specialized on spatial configurations in the 1890s. Between around 1910 and 1986 there was a nearly perfect gap in original research on configurations.

The author started this research again in the Acireale conference in 1986 organised by Mario Gionfriddo. Not only in the further Sicilian conferences of 1989, 1992, 1998, and 2004, but throughout these years the author continued this research followed by a few other mathematicians. This development and the current state of knowledge will be considered, focusing on configurations and spatial configurations.

Revisiting the Intersection Problem for Maximum Packings of K_{6n+5} with Triples

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In 1989, Gaetano Quattrocchi gave a complete solution of the intersection problem for maximum packings of K_{6n+5} with triples when the leave (a 4–cycle) is the same in each maximum packing. Quattrocchi showed that $I[2] = 2$ and $I[n] = \{0, 1, 2, \ldots, \frac{(\binom{n}{2}-4)}{3} = x\}\backslash\{x-1, x-2, x-3, x-5\}$ for all $n \equiv 5 \pmod 6 \geq 11$. We extend this result by removing the exceptions $\{x-1, x-2, x-3, x-5\}$ when the leaves are not necessarily the same. In particular, we show that $I[n] = \{0, 1, 2, \ldots, \frac{(\binom{n}{2}-4)}{3}\}$ for all $n \equiv 5 \pmod 6$.

On Some Tiling Conjectures

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Tilings and tessellations belong to the oldest structures in all mathematics. In this talk we will focus on tiling Z^n by translates of a finite set. Although this is a very special type of a tiling, it provides the simplest known counterexample to part (b) of the 18th problem of Hilbert.

We will show that various parts of mathematics, a polynomial method, Fourier analysis, algebraic geometry, provide powerful tools in the area. This approach will be illustrated by a proof of a necessary condition for the existence of a tiling obtained by Hilbert's Nullstellensatz.

On sets of type $(m,n)_2$ in $PG(3,q)$: Part One

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Let us denote by $PG(3,q)$ the three-dimensional projective space over the finite field $GF(q)$ with $q = p^h$ a prime power. A k-set of $PG(3,q)$ is a set of k points of $PG(3,q)$. We say that K is a set of type $(m,n)_2$, with $m < n$, if each plane of $PG(3,q)$ meets K in exactly m or n points; the numbers m and n are the intersection number of K with respect the planes (i.e. the subspaces of dimension two). The determination of the admissible parameters (m,n,k) of two-intersection sets in finite projective spaces is a fascinating problem which can be tackled by an interplay of number theoretic and combinatorial techniques. In [1] M. Biliotti and E. Francot studied this problem in projective planes of prime power order. A basic equation due to Tallini Scafati shows that such parameters can be expressed by the coordinates of the points of a non-singular quadric. When the difference of the intersection numbers is the order of the underlying geometry, according to the terminology introduced by T. Penttila and G.F. Royle in [2], such parameters are called *standard* and they are easily determined. In this talk we focus our attention on the non-standard case, since not much seems to be known about it.

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Resolving sets for higher dimensional projective spaces

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Let $R(n,q)$ be a resolving set for the point-hyperplane incidence graph of $\mathrm{PG}(n,q)$. In this talk estimates on the size of $R(n,q)$ are presented. We prove that if q is large enough then

$$|R(n,q)| \geq 2nq - 2\frac{n^{n-1}}{(n-2)!}.$$

This generalizes tha planar result of *Héger* and *Takáts* stating that the metric dimension of the point-line incidence graph of a projective plane of order q is $4q-4$. Translating the result of *Fancsali* and *Sziklai* about higgledy-piggledy lines to the language of resolving sets, we get that If $q = p^r$, $p > n$ and $q \geq 2n-1$ then $|R(n,q)| \leq (4n-2)q$. We prove that $|R(3,q)| \leq 8q$ and $|R(4,q)| \leq 12q$. In the cases $p < n$ and $q < 2n-1$ we prove $|R(n,q)| \leq n^2 + n - 6q$.

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On a characteristic property of the sphere

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We are studying a characteristic property of the disc in the Euclidean plane $\mathbb{R}^2$ with the help of an intersection property of the semicircles with endpoints in the boundary of the disc and a characteristic property of the ball in the Euclidean space $\mathbb{R}^3$.

Almost 2-perfect 8-cycle systems

Selda Küçükçifçi

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Let $(\mathcal{X}, \mathcal{C})$ be an 8-cycle system and let $\mathcal{C}^*$ be a collection of inside 8-cycles one from each of the cycles in $\mathcal{C}$. If $(\mathcal{X}, \mathcal{C}^*)$ is an 8-cycle system $(\mathcal{X}, \mathcal{C})$ is called almost 2-perfect. We prove that an almost 2-perfect 8-cycle system of order n exists if and only if $n \equiv 1 \pmod{16}$ and that an almost 2-perfect maximum packing of K_n with 8-cycles of order n exists for all $n \geq 8$.

Colourings of cubic graphs inducing isomorphic monochromatic subgraphs

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A *k–bisection* of a bridgeless cubic graph G is a 2–colouring of its vertex set such that the colour classes have the same cardinality and all connected components in the two subgraphs induced by the colour classes (*monochromatic components* in what follows) have order at most k. Ban and Linial conjectured that *every bridgeless cubic graph admits a 2–bisection except for the Petersen graph.* A similar problem for the edge set of cubic graphs has been studied: Wormald conjectured that *every cubic graph G with* $|E(G)| \equiv 0 \mod 2$ *has a 2–edge colouring such that the two monochromatic subgraphs are isomorphic linear forests* (i.e. a forest whose components are paths). Finally, Ando conjectured that *every cubic graph admits a bisection such that the two induced monochromatic subgraphs are isomorphic.*

In this paper, we deal with these conjectures by giving a detailed insight into the conjecture of Ban–Linial and of Wormald and provide evidence of a strong relation of both of them with Ando's conjecture. Furthermore, we also give computational and theoretical evidence in their support. As a result, we pose some open problems stronger than the above cited conjectures. Moreover, we prove Ban–Linial's conjecture for cubic cycle permutation graphs.

As a by–product, of studying 2–edge colourings of cubic graphs having linear forests as monochromatic components, we give a negative answer to a problem posed by Jackson and Wormald about certain decompositions of cubic graphs into linear forests.

On the spectrum of Octagon Quadrangle Systems of any index

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An *octagon quadrangle* is the graph consisting of a length 8 cycle $(x_1, x_2, \ldots, x_8)$ and two chords, $\{x_1, x_4\}$ and $\{x_5, x_8\}$. An *octagon quadrangle system* of order v and index λ is a pair $(X, \mathcal{B})$, where X is a finite set of v vertices and $\mathcal{B}$ is a collection of octagon quadrangles (called blocks) which partition the edge set of λK_v, with X as vertex set. We determine completely the spectrum of octagon quadrangle systems for any index λ, with the only possible exception of $v = 20$ for $\lambda = 1$.

Oddness and weak oddness of a cubic graph

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Let G be a bridgeless cubic graph. The oddness $\omega(G)$ of G is the smallest number of odd components in a 2-factor of G (i.e. a 2-regular spanning subgraph). The weak oddness $\omega'(G)$ of G is the minimum number of odd components of an even factor of G (i.e. a subgraph with all vertices of even degree). Every 2-factor is an even subgraph, but an even subgraph may contain vertices of degree 0. Then, $\omega'(G) \leq \omega(G)$ holds. In several papers, over the last few decades, weak oddness and oddness of a cubic graph appear as interchangeable definitions, implicitly assuming that they should be equal for every bridgeless cubic graph. But, the long standing discussion whether $\omega(G) = \omega'(G)$ for all bridgeless cubic graphs G was recently finished by a negative result of Lukot'ka and Mazák (see [1]). Indeed, they prove that there exists a bridgeless cubic graph with weak oddness 14 and oddness 16. Here, we improve their result showing a bridgeless cubic graph with weak oddness 6 and oddness 8.

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Orthogonal one-factorizations of complete multipartite graphs

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A *one-factor* of a graph G is a regular spanning subgraph of degree one. A *one-factorization* of G is a set $\mathcal{F} = \{F_1, F_2, \ldots, F_r\}$ of edge-disjoint one-factors such that $E(G) = \bigcup_{i=1}^{r} E(F_i)$. Two one-factorizations $\mathcal{F} = \{F_1, F_2, \ldots, F_r\}$ and $\mathcal{F}' = \{F'_1, F'_2, \ldots, F'_r\}$ are *orthogonal* if $|F_i \cap F'_j| \leq 1$ for all $1 \leq i, j \leq r$.

Let S be a set of $2n$ symbols. A *Howell design* $H(s, 2n)$ on the symbol set S is an $s \times s$ array that satisfies the following conditions:
(1) every cell is either empty or contains an unordered pair of symbols from S,
(2) every symbol of S occurs exactly once in each row and exactly once in each column of H,
(3) every unordered pair of symbols occurs in at most one cell of H.
Necessary condition for the existence of Howell designs $H(s, 2n)$ is $n \leq s \leq 2n - 1$.

A pair of orthogonal one-factorizations of an s-regular graph G on $2n$ vertices corresponds to the existence of a Howell design of type $(s, 2n)$, for which a graph G is called an *underlying graph.* If $s = n$ then a pair of orthogonal one-factorizations of a complete bipartite graph $K_{n,n}$ is equivalent to a Howell design $H_k(n, 2n)$ and moreover to a pair of mutually orthogonal latin squares of side n. In the other extreme case, an $H(2n - 1, 2n)$ is a Room square of side $2n - 1$, which corresponds to two orthogonal one-factorizations of a complete graph K_n.

An important question related to Howell designs concerns properties of graphs which are underlying graphs of Howell designs. While for $s = 2n - 1$ and $s = 2n - 2$ these graphs are unique (the complete graph K_{2n} and the cocktail party graph $K_{2n} \setminus F$, respectively, where F is a one-factor), determining these graphs in general seems to be hopeless. The goal of this talk is to show that balanced complete multipartite graphs are underlying graphs of Howell designs. The main result provides a complete solution to the existence problem of two orthogonal one-factorizations of a complete balanced multipartite graph $K_{p \times q}$.

On a method of constructing geometric designs

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The method of constructing geometric 1-designs from a matrix group that acts transitively on the set of 1-dimensional subspaces of a vector space will be introduced. It is the q-analogue of the method of constructing 1-designs from a transitive permutation group. We will analyse the possibilities for construction of q-analogues of some other structures and show some examples.

Pell Graphs

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The hypercubes Q_n are one of the most popular architecturesfor interconnection networks for multicomputers.They are highly regular, but the fact that the number of nodes grows very rapidly, as n increases,limits considerably the choice of the size of the networks.To overcome similar disadvantages some alternative architectures have been introduced.One of these alternatives is given by the *Fibonacci cubes* Γ_n [1].These graphs can be embedded in hypercubes and efficiently emulate many hypercube algorithms.Fibonacci cubes, however, turned out to be interesting by their own [4].They have a recurrent structure and several other interestingmetric, combinatorial and enumerative properties [2,5].Moreover, they also have applications in theoretical chemistry.The same can be said for many other families of graphs deriving from Fibonacci cubes, such asthe *Lucas cubes* [3],the *generalized Fibonacci cubes*,the *extended Fibonacci cubes*,the *extended Lucas cubes*,the *widened Fibonacci cubes*.In this talk we introduce the *Pell graphs* Π_n ,a new family of graphs extending Fibonacci cubesand defined in a similar way.The Fibonacci cube Γ_n is definedon the set of the binary strings of length n with no consecutive 1s,where two vertices are adjacent whenever they differ in exactly one position(i.e. when they have Hamming distance 1).Similarly, the Pell graph Π_n is defined on the set of certain stringsof length n on the alphabet $\{0, 1, 2\}$,where two vertices are adjacent whenever they differ in an elementary factor.The name of these graphs derives from the fact that the strings considered as vertices of Π_n are enumerated by the *Pell numbers*, i.e. by the numbers p_n defined by the recurrence $p_{n+2} = 2p_{n+1} + p_n$ with $p_0 = 1$ and $p_1 = 2$. Pell graphs, as well as hypercubes and Fibonacci cubes,have a recurrent structure and many other interesting structural and enumerative properties. In this talk we present some of them.

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On the extendability of particular classes of constant dimension codes

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We present a result on extendability of specific constant dimension subspace codes. An (n, M, d, k)-subspace code over $\mathbb{F}_q$ is a set of M k-dimensional subspaces of $\mathbb{F}_q^n$ having minimum distance d. We focus on large $(n, M, 2k-2, k)$-subspace codes over $\mathbb{F}_q$. A well-known upper bound on the maximum size of such a code is

$$M \leq \begin{bmatrix} n \\ 2 \end{bmatrix}_q \Big/ \begin{bmatrix} k \\ 2 \end{bmatrix}_q . \tag{1}$$

Our main result is the following.

Theorem. Let $n \equiv 0 \pmod{k}$, $(n-1) \equiv 0 \pmod{k-1}$ and $1 \leq \delta \leq (q+1)/2$. Let $\mathcal{C}$ be an $(n, M, 2k-2, k)$-code, with $M = \left[\begin{smallmatrix} n \\ 2 \end{smallmatrix}\right]_q / \left[\begin{smallmatrix} k \\ 2 \end{smallmatrix}\right]_q - \delta$. Then $\mathcal{C}$ can be extended to an $(n, \left[\begin{smallmatrix} n \\ 2 \end{smallmatrix}\right]_q / \left[\begin{smallmatrix} k \\ 2 \end{smallmatrix}\right]_q, 2k-2, k)$-code $\mathcal{C}'$.

This result implies that if no $(n, \left[\begin{smallmatrix} n \\ 2 \end{smallmatrix}\right]_q / \left[\begin{smallmatrix} k \\ 2 \end{smallmatrix}\right]_q, 2k-2, k)$-code exists, then the upper bound (1) can be improved by $(q+1)/2$.

We give an insight into the technique, coming from finite geometry, that we used to prove this result. We briefly discuss a more general extendability result on some other classes of constant dimension subspace codes whose parameters satisfy specific divisibility conditions. We finally give an application of the mentioned result.

On some self-orthogonal codes from Mathieu group M_{11}

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M_{11} is the smallest of five sporadic simple Mathieu groups and it can be represented as transitive permutation group on $11, 12, 22, 55, 66, 110, 132, 144$ and 165 points. Defining base block of a design as union of orbits of a point stabilizer acting on the set of points, we construct 1-designs, and from them codes.

Precisely, we constructed weakly self-orthogonal 1-designs,[1] and from them, by suitable extension, self-orthogonal codes. In this talk, we will present weakly self-orthogonal 1-designs and self-orthogonal codes obtained from permutation representation of M_{11} on less than 165 points (inclusive), and some of their properties. Also, we constructed self-orthogonal codes from some orbit matrices of 1-designs.

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[1] A design is weakly self-orthogonal if all the block intersection numbers have the same parity.

Equitable coloring of hypergraphs

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A hypergraph is equitably k-colorable if its vertices can be partitioned into k sets (color classes) in such a way that monochromatic edges are avoided and the number of vertices in any two color classes differs by at most one.

Such model is a direct generalization of well studied equitable graph coloring. However, the problem for hypergraphs has not been considered so far. Thus, we have established some initial complexity results. It has been proven that the problem of equitable 2-coloring of hypergraphs is NP-complete even for 3-uniform hyperstars. On the other hand for sparse systems like linear hypertrees the method of dynamic programing can be applied as long as $k \geq 2$ is fixed.

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Some conjectures on partial sums of a given set

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It is well known that difference methods have a primary role in the construction of combinatorial designs of various kinds. The continuous search for more efficient ways to use these methods often leads to intriguing problems which are very difficult despite their easy statements. Some examples are the conjectures proposed by Archdeacon et al. [1], by Buratti et al. [3] and by Meszka et al. [4]. In this talk I will present some results about the following conjecture (see [2]):

> Let $(G,+)$ be an abelian group. Let $A \neq \emptyset$ be a finite subset of $G \setminus \{0\}$ such that no 2-subset $\{x,-x\}$ is contained in A and with the property $\sum_{a\in A} a = 0$. Then there exists an ordering of the elements of A such that the partial sums are all distinct.

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The construction of combinatorial structures and linear codes from orbit matrices of strongly regular graphs

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Orbit matrices of strongly regular graphs were introduced in 2011 by M. Behbahani and C. Lam [1]. A method for constructing self-orthogonal codes from orbit matrices of strongly regular graphs admitting an automorphism group G which acts with orbits of length w, where w divides $|G|$ is given in [2]. In this talk we will present the construction of some combinatorial structures and linear codes from orbit matrices of strongly regular graphs.

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Latin squares with disjoint subsquares of two orders

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Let $n_1, \ldots, n_k \in \mathbb{Z}^+$ and $n_1 + \cdots + n_k = n$. The integer partition $(n_1, \ldots, n_k)$ of n is said to be realized if there is a latin square of order n with pairwise disjoint subsquares of order n_i for each $1 \leq i \leq k$. In this paper we construct latin squares realizing partitions of the form (a^s, b^t); that is, partitions with s parts of size a and t parts of size b, where $a < b$. Heinrich [1] showed that (1) if $s \geq 3$ and $t \geq 3$, then there is a latin square realizing (a^s, b^t), (2) (a^s, b) is realized if and only if $(s-1)a \geq b$, and (3) (a, b^t) is realized if and only if $t \geq 3$. In this talk we resolve the open cases, as outlined in a recently accepted paper [2]. We show that (a^2, b^t) is realized if and only if $t \geq 3$ and (a^s, b^2) is realized if and only if $as \geq b$.

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New strongly regular graphs from orthogonal groups $O^+(6,2)$ and $O^-(6,2)$

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The main subject of the talk is construction of strongly regular graphs from groups. In this talk, a method for constructing transitive regular graphs from finite groups will be explained. Using this method, we constructed all strongly regular graphs, with at most 600 vertices, admitting a transitive action of the orthogonal group $O^+(6,2)$ or $O^-(6,2)$. Consequently, we proved the existence of strongly regular graphs with parameters (216,40,4,8) and (540,187,58,68). In this talk the details about the obtained results will be given.

On the upper chromatic number of projective spaces

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Let $\mathcal{H} = (V(\mathcal{H}), E(\mathcal{H}))$ be a hypergraph. Several versions of colourings of hypergraphs were introduced by Voloshin, we concentrate on the upper chromatic number, see [3]. The *upper chromatic number* of $\mathcal{H}$ is the largest number of colours in a colouring without polychromatic hyperedges. A hyperedge is *polychromatic* (or rainbow) if all of its points have different colours. If B is a 2-cover of $\mathcal{H}$ (that is a set of points intersecting each hyperedge in at least two point), colouring the points of B by the same colour and all points outside B with mutually different colours, one gets a colouring with $v - |B| + 1$ colors and no polychromatic hyperedge. We call such a colouring a *trivial* one. This shows $\bar{\chi} \geq v - \tau_2 + 1$. The *decrement* of $\mathcal{H}$ is the quantity $\mathrm{dec}(\mathcal{H}) := v - \bar{\chi}(\mathcal{H})$.

For projective planes (the hyperedges are just lines) the first important results are due to Bacsó and Tuza. Then Bacsó, Héger and Szőnyi [1] gave estimates on the upper chromatic number of projective planes. In some cases they showed that the upper chromatic number comes from a trivial colouring where B is the smallest 2-cover.

The aim of the present talk is to generalize these results to projective spaces and to show a stability result for colourings using almost as many colours as the upper chromatic number. To be more precise we fix a k and consider the hypergraph $\mathrm{PG}_k(n, q)$ on the points of the projective space $\mathrm{PG}(n, q)$, whose hyperedges are the k-dimensional subspaces. In this case 2-covers are just 2-fold blocking sets with respect to k-dimensional subspaces (commonly called 2-fold $(n-k)$-blocking sets in finite geometry). We shall focus on the case $2k > n$. The smallest such set is the union of two (disjoint) $(n-k)$-dimensional subspaces, so we shall show that

$$\bar{\chi}(\mathrm{PG}_k(n, q)) = \frac{(q^{n+1}-1)}{q-1} - 2\frac{(q^{n-k+1}-1)}{(q-1)} + 1.$$

The stability version says the following: if there is no polychromatic hyperedge in a colouring with decrement at most $(2+c)\frac{(q^{n-k+1}-1)}{(q-1)}$, where $c > 0$, then the colouring has to be trivial. So far, we have roughly $\frac{1}{17}$ for the value of c. Actually, there is a bound on the characteristic p, depending on c. Note that in many cases we have different 2-fold $(n-k)$-blocking sets not just the union of two $(n-k)$-subspaces. The proof uses the methods of Ferret, Storme, Sziklai and Weiner [2]. In the particular cases when $q = p$ or p^2, where p is a prime, Ferret, Storme, Sziklai and Weiner, and Blázsik in his Masters thesis proved stability type results for 2-fold $(n-k)$-blocking sets. Using these results, we managed to improve the constant c to be roughly $\frac{1}{5}$ in these cases.

This research was supported by the Slovenian-Hungarian ARRS-OTKA Grant NN 114614 (in Hungary) and N1-0032 (in Slovenia).

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On a generalization of complete mappings

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Given a group G of order n, we denote by $\mathcal{D}(G)$ the set of the element-orders of G, and we say that $\alpha : \mathcal{D}(G) \to \mathbb{N} \cup \{0\}$ is admissible whenever $\sum_{d\in\mathcal{D}(G)} \alpha(d) = n$. An *$\alpha$-complete mapping of G* is a permutation Ψ of G such that

$$|\{g \in G \mid \Psi(g) - g \text{ has order } d\}| = \alpha(d) \qquad \text{for every } d \in \mathcal{D}(G).$$

In other words, Ψ is an α-complete mapping of G if the function $\Psi - id$ maps exactly $\alpha(d)$ elements of G into elements of order d, for every $d \in \mathcal{D}(G)$. For example, let $G = \mathbb{Z}_{15}$ and let $\alpha : \mathcal{D}(G) = \{1, 3, 5, 15\} \to \mathbb{N} \cup \{0\}$ be the map such that $\alpha(1) = \alpha(3) = \alpha(15) = 1$ and $\alpha(5) = 12$. The reader may have fun checking that the permutation $\Psi = (1)(3, 8, 13, 11, 6)(0, 5, 10)(9, 14, 4)(12, 2, 7)$ is an α-complete mapping. We point out that a classic complete mapping of G is α-complete, where $\alpha(d)$ counts the number of elements of order d in G.

We consider the problem of determining the existence of an α-complete mapping of G for every admissible function α, and show that they can be used to construct 2-factorizations of the complete (equipartite) graph into copies of t distinct “uniform” 2-factors.

Graceful polynomials of small degree

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Starting from a graph we can construct a family of polynomials in $\mathbf{Z}_2[x]$ – one for every degree – with as many variables as the number of vertices. We call them "graceful polynomials" because in some cases they are efficient tools for proving that the graph is non-graceful. They generalise Rosa's polynomial (see [2]). In [3] such polynomials were applied to graceful trees. In this talk we classify graphs whose graceful polynomial vanishes, for small degrees up to 4. Some families of non-graceful graphs emerge from the discussion.

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High Rate Low Density Parity Check Codes from Difference Covering Arrays

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This presentation will suggest a combinatorial construction of low-density parity-check (LDPC) codes from difference covering arrays. While the original construction by Gallagher was by randomly allocating bits in a sparse parity-check matrix, over the past 20 years researchers have used a variety of more structured approaches to construct these codes, with the more recent constructions of well-structured LDPC coming from balanced incomplete block designs (BIBDs) and from Latin squares over finite fields. However these constructions have suffered from the limited orders for which these designs exist. Here we present a construction of LDPC codes of length $4n^2 - 2n$ for all n using the cyclic group of order $2n$. These codes achieve high information rate (greater than 0.8) for $n = 8$, have girth at least 6 and have minimum distance 6 for n odd.

On sets of type $(m, n)_2$ in $PG(3, q)$: Part Two

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Let $PG(3, q)$ be the projective space of dimension three and order q, with $q = p^h$ a prime power. A set K of type $(m, n)_2$ in $PG(3, q)$ is a set of points such that every plane contains either m or n points of K. the numbers m and n are the intersection number of K with respect the planes. When the difference of the intersection numbers is the order of the underlying geometry, i.e. $n = m + q$, according to the terminology introduced by T. Penttila and G.F. Royle in [1], such parameters are called *standard*. By standard equations, it is easy to verify that a set of type $(m, m + q)_2$ in $PG(3, q)$ has two possible sizes either $|K| = h_m(q) = m(q + 1)$ or $|K| = k_m(q) = (m + q)(q^2 + 1)/(q + 1)$, see [2]. The classical example of such a set is a partial spread of size m; however, several other families are known. In this talk we present some properties and their classification for small parameters.

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Acknowledgments

Acknowledgments

The members of the Organizing Committee of HyGraDe 2017 would like to express their cordial thanks to all who helped make this conference happen.

First of all, we are very grateful to our invited speakers who have accepted our invitation to give a plenary talk to this congress, making it a recognized international scientific event. Next, we express our gratitude to our participants whose presence and contributed talk made this scientific event successful.

We have much pleasure in thanking several organizations, in particular, the Universities of Catania, Messina and Naples "Federico II", and the "Istituto Nazionale di Alta Matematica - INdAM - sezione GNSAGA", whose financial contribution was fundamental to organize the congress. We also thank the Departments of Mathematics and Informatics (University of Catania) and the Department MIFT (University of Messina) for their financial contribution and for covering the expenses of the Inaugural Celebration. Several people from the above mentioned institutions deserve special recognition due to their role in the administrative organisation of the congress HyGraDe 2017: Rossella Baldoni and Grazia Strano (Catania), Antonio Denaro, Marco Noli Maio and Santa Bardetta (Messina), Carmela Ippolito, Deborah Tomasello and Claudio Roberti (Naples).

We express our gratitude to Prof. Paolo Giaquinta, as Dean of Accademia Peloritana dei Pericolanti, and Prof. Dragan Marušič, as Editor-in-chief of the journal "Art of Discrete and Applied Mathematics - ADAM" for giving us the possibility to host the Conference Proceedings on their journals.

Our very special thanks go to: the staff of the Elihotel, the host of the Conference, and especially to Santino Altadonna (director of the Hotel) for their support and excellent work; Nancy Pandolfino, Congress Organizer of "Lisciotto Turismo SRL", for taking care of many issues in the organisation; Anna Cerere and Giuseppe Rubino, owners of "PubbliVisual SRL", for their careful work on the congress material arranged for the participants; Francesco Furnari, for arranging all transfers of our invited speakers.

The organizers gladly acknowledge also: Giuseppe Izzo (University of Naples "Federico II") for installing OCS (Open Conference System) on the department server and his help in improving the presentation of the conference website; and Guglielmo Lunardon and Laura Bader for their support in the organisation of HyGraDe 2017 by "Associazione di Volontariato Combinatorics2014".

The Conference Organizers
Francesco Belardo and Giovanni Lo Faro

List of Talks

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