Vertex-transitive-odd numbers

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A 6-cycle system (X, C) is said to be almost 2-perfect if it is possible to place a 6-cycle inside each 6-cycle in C so that the resulting collection of 6-cycles is a 6-cycle system. Not too surprisingly sometimes this is possible and sometimes its not. For example the 6-cycle system (X, C_1) is almost 2-perfect.

$$X = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$$

$$C_1 = \begin{cases} (0,1,2,3,4,5) & \to (0,2,4,1,5,3) \\ (0,2,4,1,6,7) & \to (0,4,6,2,7,1) \\ (0,3,7,8,4,6) & \to (0,7,4,3,6,8) \\ (0,8,6,5,7,4) & \to (0,6,7,8,4,5) \\ (1,7,2,8,5,3) & \to (1,2,5,7,3,8) \\ (1,8,3,6,2,5) & \to (1,3,2,8,5,6) \end{cases}$$

The 6-cycle system (X, C_2) is *not*!

$$X = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$$

$$C_2 = \begin{cases} (0,1,2,3,4,5) \\ (0,2,4,1,3,6) \\ (0,3,5,1,7,8) \\ (0,4,6,8,2,7) \\ (1,6,5,7,3,8) \\ (2,5,8,4,7,6) \end{cases}$$

This is an elementary survey showing that the spectrum for almost 2-perfect 6-cycle systems is the set of all $n \equiv 1$ or 9 (mod 12), (= the spectrum for 6-cycle systems). This can be extended to maximum packings.