Mixed hypergraphs and beyond

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The concept of *mixed hypergraph* coloring was introduced by Voloshin in 1993. Mixed hypergraphs are composed of so-called C-edges and D-edges, with the requirement that every feasible coloring assigns a common color to at least two vertices of each C-edge, and distinct colors to at least two vertices of each D-edge.

Generalizing this structure class, a decade ago Bujtás and Tuza introduced *stably bounded hypergraphs*, each edge E of which may have four restrictions s, t, a, b with the following meaning:

- E contains vertices of at least s distinct colors;
- E contains vertices of at most t distinct colors;
- some color occurs in E at least a times;
- each color occurs in E at most b times.

Hence, inside each edge E, the values s and t bound the number of colors, while a and b bound the maximum multiplicity of colors. In the general setting, the values s, t, a, b may be non-uniform over the edges, i.e. they are four functions from the edge set to the set of positive integers. The theory of these kinds of coloring is quite developed, but still there are several challenging problems and conjectures which remained open since many years.

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