On finite p-groups with subgroups of breadth 1 — Corrigendum

GIOVANNI CUTOLO, HOWARD SMITH AND JAMES WIEGOLD

Lemma 2.8 as published is not correct, in that some conclusions drawn in it require the hypothesis that the subgroup C is not abelian. A correct version of the lemma is the following:

Lemma 2.8. *Let G be a finite 2-group of s-breadth 1, and assume that G has no dihedral subgroup of order eight. Let a be a noncentral involution in G and let $b \in G \setminus C_G(a)$. If $H = \langle a, b\rangle$ and $C = C_G(H)$ then $|C/Z(C)| \leq 4$ and b^2 is not a square in C. Moreover, $V := \langle a, a^b\rangle$ is noncyclic of order four and:*

(i) *if C is not abelian then $H = V \rtimes \langle b\rangle$, $\langle b^2\rangle \lhd G$ and all elements of $C \setminus Z(C)$ have order four;*
(ii) *if C is hamiltonian then b has order four and $C' = H' = \langle aa^b\rangle$;*
(iii) *if C is not a Dedekind group then b has order eight and $C' = \langle aa^b b^4\rangle$.*

The proof is as in the paper, but a reference to Corollary 2.4 must be anticipated, in order to show that if a normalises $\langle b\rangle$ then C is abelian. The argument is as follows. Suppose that a normalises $\langle b\rangle$. Then $[a, b]$ is a square in $\langle b\rangle$, hence in H. By Corollary 2.4, this implies that C is a Dedekind group. As remarked in the first lines of the proof, $[a, b^2] = 1$ (because a has breadth 1) and so $b^2 \in Z(C)$. Now, $H = \langle b\rangle \rtimes \langle a\rangle$ and b cannot have order 4, otherwise $\langle a, b\rangle \simeq D_8$. Therefore b^2 is an element of $Z(C)$ of order greater than 2, and it follows that C is abelian. The conclusion is that if C is not abelian then $\langle b\rangle \neq \langle b\rangle^a$; as in the proof in the paper this means that $\langle b\rangle_G = \langle b^2\rangle$, hence $\langle b^2\rangle \lhd G$. Once this has been observed, the proof goes exactly as in the paper.

The proof of Lemma 2.9 needs a corresponding little fix. Namely, in the first paragraph, the reduction to the case when C is not abelian must precede, and not follow, the choice of the element $x \in N_{C_a}(\langle b\rangle)$, so a portion of text preceding the phrase "Suppose first that C is abelian" must be moved after this reduction has been made. Explicitly, the following change gives the desired result:

- replace "Moreover $a \notin N_G(\langle b\rangle)$, so there exists $x \in N_{C_a}(\langle b\rangle) \setminus C_b$; note that $G = C\langle a, b, x\rangle$. Also note that since $[b, x] \in \langle b\rangle$ while $u = aa^b \notin \langle b\rangle$ and $|G'| = 4$ we must have $[b, x] = b_0$ and $G' = \langle u\rangle \times \langle b_0\rangle$, where b_0 generates the socle of $\langle b\rangle$" with "There exists $x \in C_a \setminus \langle a\rangle C$; note that $G = C\langle a, b, x\rangle$"
- after "we may assume that C is not abelian." add "Then $a \notin N_G(\langle b\rangle)$, so we may assume that x has been chosen in $N_G(\langle b\rangle)$. Then, since $[b, x] \in \langle b\rangle$ while $u = aa^b \notin \langle b\rangle$ and $|G'| = 4$ we must have $[b, x] = b_0$ and $G' = \langle u\rangle \times \langle b_0\rangle$, where b_0 generates the socle of $\langle b\rangle$."