DISTRIBUTED RESOURCES RESERVATION ALGORITHM FOR GRID NETWORKS

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PROBLEM FORMALIZATION

1. Let graph $\mathbf{G_N=(E_N,V_N,I_N,J_N)}$ - represents network
   - $\mathbf{E_N}$ – is set of physical network connections
   - $\mathbf{V_N}$ – is set of computational nodes (processors)
   - $\mathbf{I_N}$ – is set of weights, represented network connection bandwidth in Mbps
   - $\mathbf{J_N}$ – is set of weights, represented productivity of processors in MIPS or MFLOPS
2. Let graph $\mathbf{G_T=(E_T,V_T,I_T,J_T)}$ – represents distributed application requirements to resources
   - $\mathbf{E_T}$ – set of application network communications
   - $\mathbf{V_T}$ – set of application computational processes
   - $\mathbf{I_T}$ – requirements to bandwidth in Mbps
   - $\mathbf{J_T}$ – requirements to productivity of processors in MIPS or MFLOPS

DISTRIBUTED RESOURCES RESERVATION ALGORITHM

- Matrix of possible superpositions proposed
- Conditions based on wave decomposition of graphs used to reduce complexity of combinatorial part of algorithm

MATRIX OF POSSIBLE SUPERPOSITIONS

It is binary table $\mathbf{M_{i,j}}$ and each cell represents whenever it possible or not to superpose pair of vertices $\mathbf{V_{N,i}}$ and $\mathbf{V_{T,j}}$. The values of **M** are generated depending on preliminary tests performing on both graphs.

- $\mathbf{M_{i,j}}$ has value **"true"** if there are no restrictions to try superpose two vertices $\mathbf{V_{N,i}}$ and $\mathbf{V_{T,j}}$ based on preliminary tests;
- $\mathbf{M_{i,j}}$ has value **"false"** another way.

PRELIMINARY PART OF ALGORITHM

Conditions that form matrix of possible superpositions:

Let $|\mathbf{V_x}|$ be number of edges connected to vertex $\mathbf{V_X}$.

$$|\mathbf{V_{N,i}}| < |\mathbf{V_{T,j}}| \rightarrow \mathbf{M_{i,j}=false} \qquad \mathbf{(1)}$$

$$\mathbf{J_{N,i}} < \mathbf{J_{T,j}} \rightarrow \mathbf{M_{i,j}=false} \qquad \mathbf{(2)}$$

Let $\mathbf{Vin_{N,i}}$ be subset that consists from incoming edges that are incident to vertex $\mathbf{V_i}$

$$|\mathbf{Vin_{N,i}}| < |\mathbf{Vin_{T,j}}| \rightarrow \mathbf{M_{i,j}=false} \qquad \mathbf{(3)}$$

Let $\mathbf{Iin_{N,i}}$ be subset of weights that corresponds to $\mathbf{Vin_{N,i}}$

$$\sum \mathbf{Iin_{N,i}} < \sum \mathbf{Iin_{T,j}} \rightarrow \mathbf{M_{i,j}=false} \qquad \mathbf{(4)}$$

PRELIMINARY PART OF ALGORITHM

Let $\mathbf{Vout_{N,i}}$ be subset that consists from outgoing edges that are incident to vertex $\mathbf{V_i}$

$$|\mathbf{Vout_{N,i}}| < |\mathbf{Vout_{T,j}}| \rightarrow \mathbf{M_{i,j}=false} \quad \mathbf{(5)}$$

Let $\mathbf{Iout_{N,i}}$ be subset of weights that corresponds to $\mathbf{Vout_{N,i}}$

$$\sum \mathbf{Iout_{N,i}} < \sum \mathbf{Iout_{T,j}} \rightarrow \mathbf{M_{i,j}=false} \quad \mathbf{(6)}$$

ADDITIONAL TESTS

Let $\mathbf{W_{X,Y,k}}$ be a set of vertices in wave expansion of graph **X** starting from vertex **Y** with maximal edge distance between vertices **k**

$$|\mathbf{W_{N,i,k}}| < |\mathbf{W_{T,j,k}}| \rightarrow \mathbf{M_{i,j}=false, where\ k=1..|W_{T,j}|}\ \mathbf{(7)}$$

Let $\mathbf{U_{X,Y,k}}$ be a set of vertices weights that corresponds to vertices in $\mathbf{W_{X,Y,k}}$

$$\sum \mathbf{U_{N,i,k}} < \sum \mathbf{U_{T,j,k}} \rightarrow \mathbf{M_{i,j}=false, where\ k=1..|W_{T,j}|}\ \mathbf{(8)}$$

Let $\mathbf{Q_{X,Y,k}}$ be a subset of edges that are part of wave expansion of graph **X** starting from vertex **Y** with maximal edge distance between vertices **k**

$$|\mathbf{Q_{N,i,k}}| < |\mathbf{Q_{T,j,k}}| \rightarrow \mathbf{M_{i,j}=false, where\ k=1..|Q_{T,j}|}\ \mathbf{(9)}$$

Let $\mathbf{R_{X,Y,k}}$ be a set of edges weights that corresponds to edges in set $\mathbf{Q_{X,Y,k}}$

$$\sum \mathbf{R_{N,i,k}} < \sum \mathbf{R_{T,j,k}} \rightarrow \mathbf{M_{i,j}=false, where\ k=1..|Q_{T,j}}\ \mathbf{(10)}$$

VERTEX SORTING

Let $\mathbf{T_{T,i}}$ represent number of edges that are incident to vertices that have index lower then **i**;

Let $\mathbf{P_{T,i}}$ – the number of possible substitutions available for vertex $\mathbf{V_{T,i}}$ of graph $\mathbf{G_T}$:

$$\mathbf{V_{T,i} \leftarrow V_{T,k}, \text{ where } T_{T,k}=\min(T_{T,j}) \text{ for } j=(i+1)..|V_T|} \quad \textbf{(11)}$$

In case when $\mathbf{T_{T,i}=T_{T,k}}$ another condition applied:

$$\mathbf{V_{T,i} \leftarrow V_{T,k}, \text{ where } P_{T,k}=\min(P_{T,i},P_{T,j})} \quad \textbf{(12)}$$

COMBINATORIAL PART OF ALGORITHM

Conditions of combinatorial part:

$$\mathbf{M_{i,j}=true} \qquad \mathbf{(13)}$$

Let $\mathbf{T_{N,i}}$ be number of edges that incident to vertex $\mathbf{V_{N,i}}$ and to some vertex that belongs to the partial substitution $\boldsymbol{\varphi_i(s)}$

$$\mathbf{T_{N,i} \geq T_{T,j} (14)}$$

$$\mathbf{(v_i,v_k) = (\varphi_i(v_i), \varphi_i(v_k)), \text{ where } k=1..i} \qquad \mathbf{(15)}$$

$$\mathbf{(i_i,i_k) \geq (\varphi_i(i_k), \varphi_i(i_k)), \text{where } k=1..i} \qquad \mathbf{(16)}$$

PRODUCTIVITY RESEARCH OF ALGORITHM

Associated weights:

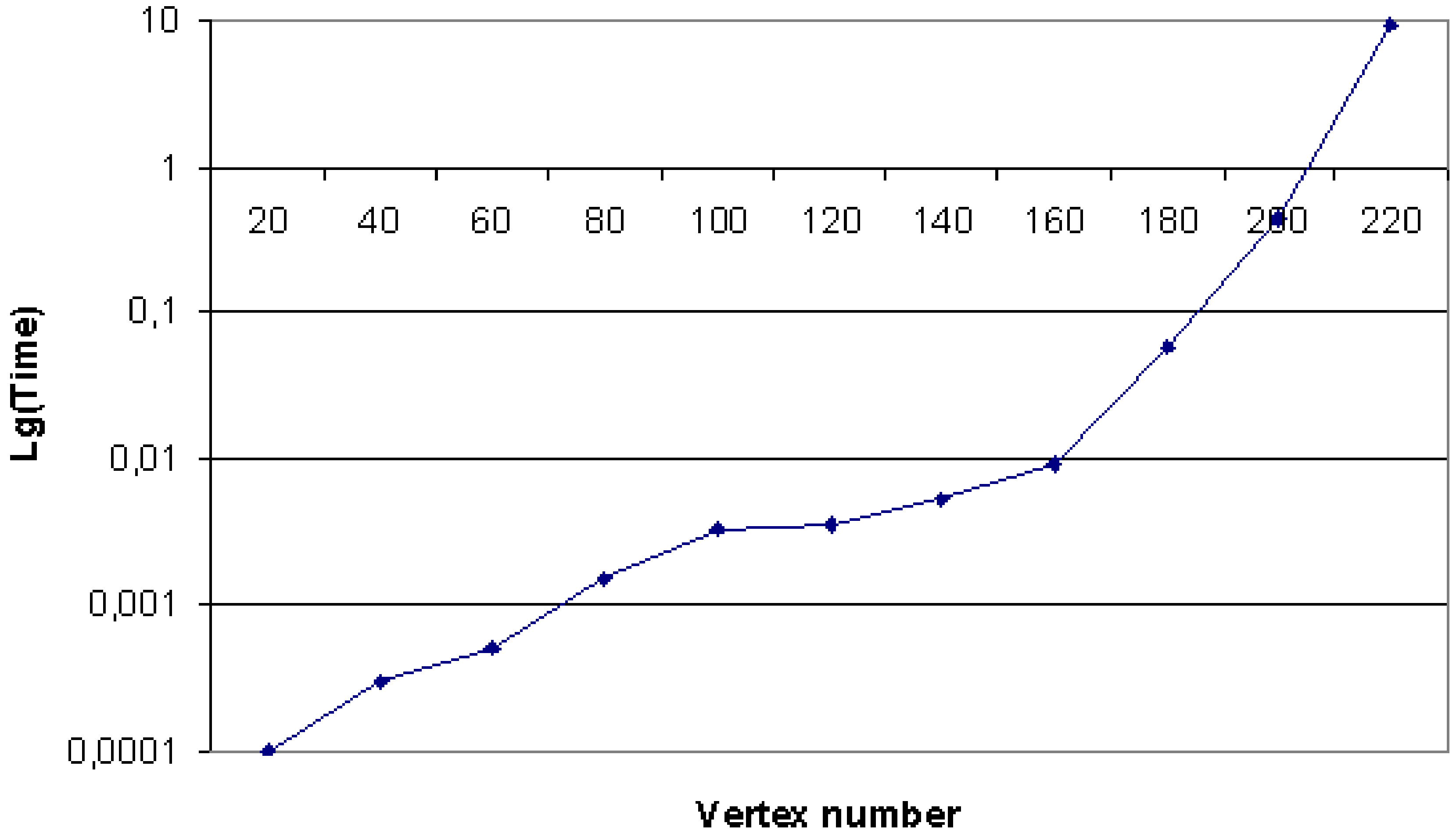
$$10 \leq I_N \leq 100 \text{ (Mbps)}$$

$$200 \leq J_N \leq 3000 \text{ (MIPS/MHz)}$$

Graphs reduction factors:

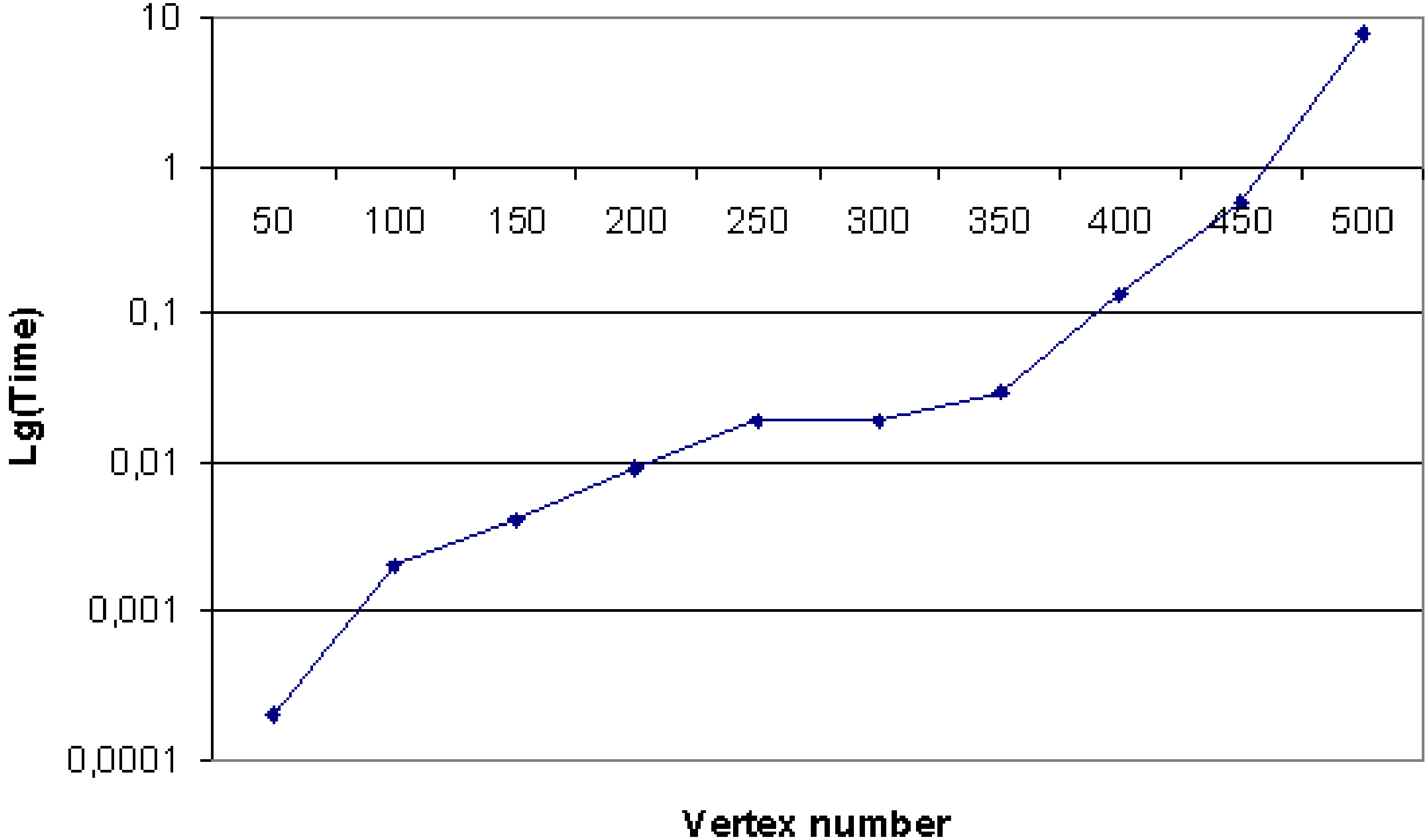
- k_{nv} – vertex reduction factor ($|V_T| = k_{nv}|V_N|$);
- k_r – edges reduction factor ($|E_T| = k_r|E_N|$);
- k_W – weights reduction factor ($J_{T,i} = k_w J_{N,i}$, $I_{T,i,j} = k_w I_{N,i,j}$).

AVERAGE RESERVATION TIME



Parameters: k_{nv}=0.2, k_r=0.5, k_w=0.2, network graphs edges density 50%

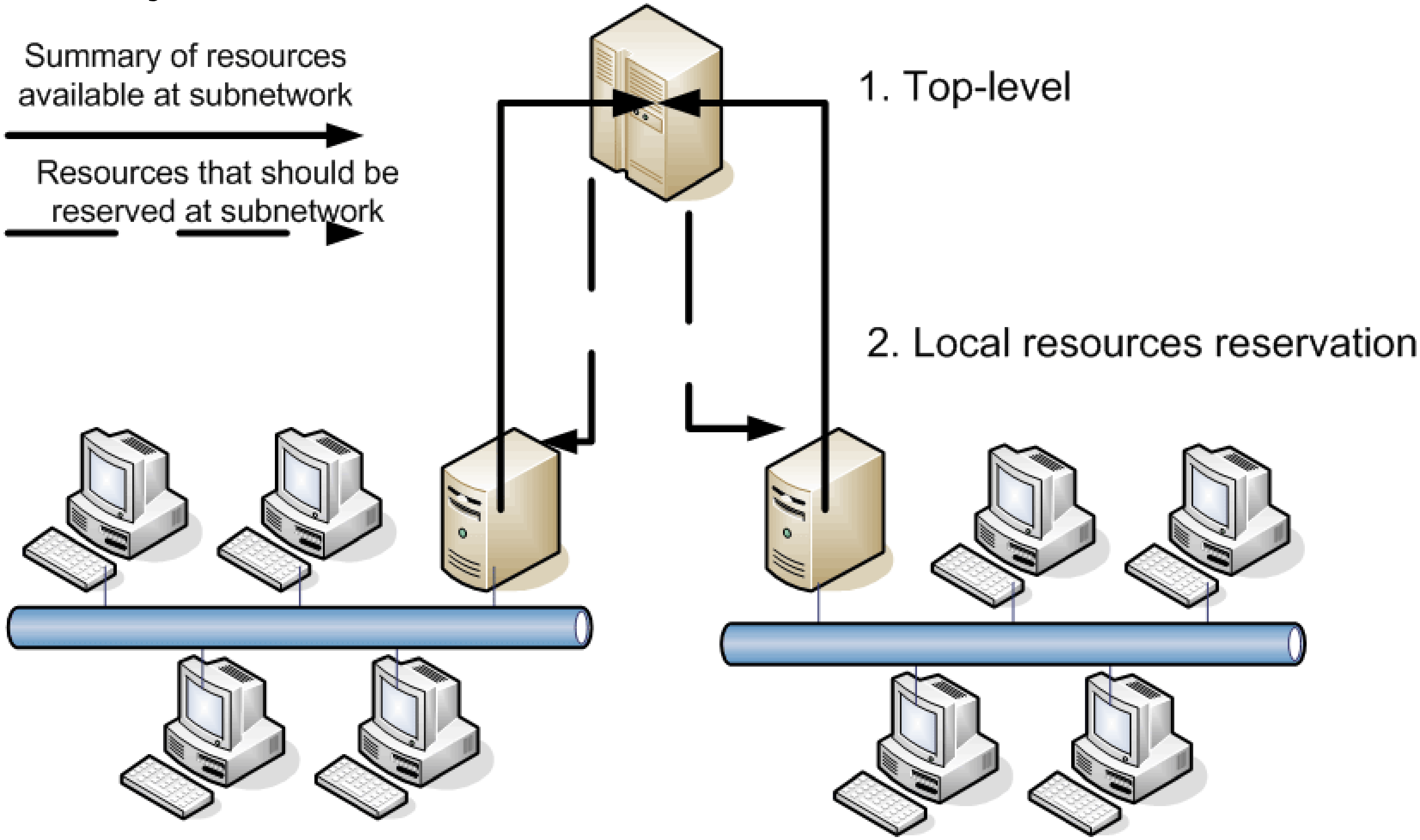
AVERAGE RESERVATION TIME



Parameters: k_{nv}=0.1, k_r=0.5, k_w=0.2, network graphs edges density 30%

VERY LARGE NETWORKS

Multi-layer resources reservation method:



CONCLUSIONS

- New distributed resources reservation algorithm presented;
- Matrix of possible superpositions proposed;
- Conditions based on wave decomposition of graphs designed and used to reduce complexity of combinatorial part of algorithm;
- Productivity of algoritms researched on different graphs;
- Multi-layer method proposed for very large networks.