Solving linear systems of the form $(A + \gamma UU^T)\mathbf{x} = \mathbf{b}$

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We consider the solution of large linear systems of the form

$$(A + \gamma UU^T)\mathbf{x} = \mathbf{b}$$

with A of size $n \times n$, $\gamma > 0$ a parameter and U of size $n \times k$, by preconditioned Krylov subspace methods. We make the following assumptions:

- A is possibly singular, but $A + \gamma UU^T$ is nonsingular;
- forming $A + \gamma UU^T$ explicitly is undesirable, for example due to loss of structure or sparsity in A;
- the dimension k is much smaller than n, but not small.

Linear systems of this form arise in several areas of scientific computing, including the solution of the Stokes and Navier-Stokes problems with augmented Lagrangian methods, the solution of reduced KKT systems in constrained optimization, the discretization of certain integro-differential equations, the solution of PDEs with non-local boundary conditions, and elsewhere. Solving such linear systems can be challenging, especially for large values of the parameter γ. We will present and investigate different variants of a preconditioning technique based on a suitable splitting of the coefficient matrix. The performance of these preconditioners will be illustrated by means of numerical experiments on a variety of test problems.

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