Linear algebra of HP-splines

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Exponential-polynomial splines are a natural generalization of polynomial splines important in several applications ranging from geometric modeling to image analysis passing through isogeometric analysis and system theory. They are piecewise-defined functions consisting of segments belonging to the null space of a differential operator $L_\ell = (\mathcal{D} + \alpha\mathcal{I})^\ell$, where $\alpha \in \mathbb{C}$, $\mathcal{D}$ is the first derivative operator, $\mathcal{I}$ is the identity operator and $\ell \in \mathbb{N}$. In case $\alpha \in \mathbb{R}$, they are called *hyperbolic-polynomial splines*.

In this talk, we discuss the properties of the family of hyperbolic-polynomial penalized splines, named HP-splines, proposed in [1] obtained through the solution of a linear system that strongly depends on α. The parameter α plays an important role also on the conditioning affecting the constructive method. A theoretical bound for the condition number of the matrix defining the linear system is given, based on classical results of generalized singular values decomposition. Reproduction of the exponential space $\mathbb{E}_{2,\alpha} = \{e^{-\alpha x},\ xe^{-\alpha x}\}$ is also proved, together with the first and second moments, which find an interesting counterpart in statistics, generalizing the inspiring results valid for the analog polynomial P-splines [2].

Joint work with C. Conti

References

[1] R. Campagna and C. Conti, *Penalized hyperbolic-polynomial splines*, Applied Mathematics Letters, Volume 118 (2021).

[2] P. H. C. Eilers and B. D. Marx, *Flexible smoothing with B-splines and penalties*, Statistical Science, Volume 11(2) (1996).