Geometrical properties of the graph p-Laplacian spectrum

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The spectrum of the graph p-Laplacian operator, including the limit cases $p = 1$ and $p = \infty$, has tight relations with different geometrical properties of the graph itself. The spectrum of the 1-Laplacian has been proved to provide approximations of the Cheeger cuts of the graph. Similarly, the spectrum of the discrete ∞-Laplacian is related to the maximal radius that allows to inscribe a fixed number of disjoint balls in the graph. Particulary significative in these approximations is the number of nodal domains induced by the corresponding eigenfunctions. In this work we present new upper and lower bounds for this number. Moreover, we provide a characterization of the graph ∞-Laplacian eigenvalues and properties of its eigenfunctions in terms of extremal points of a class of functionals involving only weighted linear Laplacians. We compare our results with the analogous counterparts in the continuum case and use them to define efficient numerical methods to compute the partial eigenspectrum of the graph p-Laplacian, including the limit cases.

Joint work with M. Putti, F. Tudisco, M. Burger, N. Segala

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