Deviation maximization for rank-deficient problems

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The problem of finding a rank-revealing QR (RRQR) decomposition of a matrix A is nowadays a classic problem in numerical linear algebra. RRQR factorization has been introduced [2] for least squares problems where the matrix has not full column rank: in such a case, a plain QR computation may lead to an R factor in which the number of nonzeros on the diagonal does not equal the rank and the matrix Q does not reveal the range nor the null space of the original matrix. Since the QR factorization is essentially unique once the column ordering is fixed, these techniques all amount to finding an appropriate column permutation. In this talk we introduce a recent block pivoting technique we called "Deviation Maximization" [1] that is based on correlation analysis, and apply it to compute RRQR as an alternative to the well known block version of the QR factorization with the column pivoting method [3], currently implemented in the `xgeqp3` subroutine of LAPACK. We show that the resulting algorithm, named QRDM, has similar rank-revealing properties of QP3 and shorter execution times. We present numerical results on a wide data set of numerically singular matrices.

Joint work with F. Marcuzzi

References

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[2] G. Golub and V. Klema and G.W. Stewart, *Rank degeneracy and least squares problems*, Technical report STAN-CS-76-559, Department of Computer Science, Stanford University (1976).

[3] G. Quintana-Ortì and X. Sun and H. Bischof, *A BLAS-3 Version of the QR Factorization with Column Pivoting*, SIAM Journal on Scientific Computing, 19, n.5 (1998), pp. 1486–1494.