Remember where you came from: Hitting times for second-order random walks

Dario Fasino
University of Udine, Udine (Italy)
dario.fasino@uniud.it

Effective mathematical models for navigation and diffusion processes in complex networks are often based on random walks, i.e., Markov chains $\{X_n\}$ with a stochastic matrix P describing edge transition probabilities,

$$\mathbf{P}(X_{n+1} = j | X_n = i) = p_{ij}.$$

This idea has recently been extended by accounting for an earlier step using, e.g., non-backtracking random walks or other network navigation strategies encountered in machine learning algorithms. The transitions of the resulting stochastic processes are described by formulas such as the following:

$$\mathbf{P}(X_{n+1} = k | X_n = j, X_{n-1} = i) = p_{ijk}.$$

Such processes can be naturally recast as Markov chains on the edges of the original network. Following this approach, non only stationary densities but also mean hitting times and return times can be defined for these second-order random walks.

In the present work, we investigate the problem of computing these second-order mean hitting and return times and analyze their relations with the stationary density of the process. One of our guiding questions is the following: To what extent do known facts in Markov chain theory (e.g., Kac's lemma and Kemeny's constant) carry over to second-order random walks?

Joint work with A. Tonetto *and* F. Tudisco

References

[1] D. Fasino, A. Tonetto, and F. Tudisco, *Hitting times for non-backtracking random walks*, arXiv:2105.14438, 2021.