Distance to singularity for quadratic matrix polynomials

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Given a regular matrix polynomial, an interesting problem consists in the computation of the nearest singular matrix polynomial, which determines its distance to singularity. When this distance is small, the conditioning of the eigenvalues is affected by small perturbations.

In this talk we focus - for simplicity - on the case of a regular quadratic matrix polynomial $\lambda^2 A + \lambda B + C$ with $A, B, C \in \mathbb{C}^{n\times n}$ and look for the nearest (with respect to the Frobenius norm) singular quadratic matrix polynomial $\lambda^2(A+\Delta A)+\lambda(B+\Delta B)+(C+\Delta C)$. To do this we generalize the idea presented in [1] for pencils, by imposing that the determinant of the perturbed matrix polynomial nihilates on a set of complex points $\{\mu_i\}_{i=1}^N$ with $N > 2n$, which implies that the determinant vanishes identically. We proceed by successively minimizing, with respect to perturbations of fixed norm $\|[\Delta A, \Delta B, \Delta C]\|_F = \varepsilon$, the functional

$$F_\varepsilon(\Delta A, \Delta B, \Delta C) = \frac{1}{2}\sum_{i=1}^{N} \sigma_{\min}^2\left(\mu_i^2(A+\Delta A)+\mu_i(B+\Delta B)+(C+\Delta C)\right)$$

where $\sigma_{\min}$ denotes the smallest singular value. We do the minimization by integrating the gradient system associated with the functional. In addition, to drive the functional to zero, we tune the norm value ε by an iterative method. Our technique also applies to structured problems, for example to palindromic polynomials, by projecting the gradient onto the structure. Illustrative examples will be shown during the talk.

Joint work with N. Guglielmi

References

[1] N. Guglielmi, C. Lubich, and V. Mehrmann, *On the nearest singular matrix pencil*, SIAM Journal on Matrix Analysis and Applications, 38 (2017), pp. 776–806.