Improving AMG interpolation through energy minimization

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Algebraic Multigrid (AMG) is a very popular iterative method in several applications due to its effectiveness in solving linear systems arising from the discretization of PDEs. The key feature of AMG is its optimality that is its ability to guarantee, at least in some problems, a convergence rate that is independent of the mesh size. This feature is obtained through a good interplay between the smoother and the interpolation. Unfortunately, for difficult problems such as those arising from linear elasticity or diffusion problems with strong contrasts in the coefficients, standard smoothers and interpolation techniques are not enough to ensure fast convergence. In these cases, improving the prolongation operator by minimizing its energy may greatly enhance AMG convergence [1,2]. In this talk, we show how energy minimization can be seen as a constrained minimization problem, where the constraint is twofold: the prolongation must be sparse, and its range must contain the operator near-kernel. To solve this problem, we propose an iterative algorithm based on the null-space method and preconditioning to speed-up the set-up time. Finally, thanks to some numerical experiments we demonstrate how the convergence rate can be significantly increased at a reasonable set-up cost.

Joint work with A. Franceschini *and* G. Isotton

References

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[2] Manteuffel, T. A., Olson, L. N., Schroder, J. B. and Southworth, B. S., *A Root-Node-Based Algebraic Multigrid Method*, SIAM Journal on Scientific Computing, 39 (2017), pp. S723–S756.