Relative error propagation in linear ordinary differential equations: long-time behavior of condition numbers

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We consider the propagation, along the solution of a linear ordinary differential equation $y'(t) = Ay(t)$, of perturbations in the initial value $y(0) = y_0$. Of course, this propagation is well understood when the perturbations are measured by absolute errors. In this talk, we analyze the propagation when the perturbations are measured by relative errors, rather than absolute errors. In other words, we are interested in the relative conditioning of the linear problem $y_0 \mapsto \mathrm{e}^{tA}y_0$, i.e. the conditioning of the action of the matrix exponential e^{tA} on a vector. Our analysis is a qualitative study of the long-time behavior of this conditioning. We introduce three condition numbers: the first is relevant to a fixed direction of perturbation, the second considers the worst case for the direction of perturbation and the third considers the worst case for the direction of perturbation and the initial value. The long-time behavior of these three condition numbers is studied. We discuss in which manner a strong nonnormality of the matrix A can affect this long-time behavior and its onset. This study extends the previous work [1], where only normal matrices A were considered.

Joint work with A. Farooq

References

[1] S. Maset, *Conditioning and relative error propagation in linear autonomous ordinary differential equations*, Discrete and Continuous Dynamical Systems Series B, vol. 23 (7) (2018), pp. 2879–2909.