

Mixed precision recursive block diagonalization for bivariate functions of matrices

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Various numerical linear algebra problems can be formulated as evaluating bivariate function of matrices [1]. The most notable examples are the Fréchet derivative along a direction, the evaluation of (univariate) functions of Kronecker-sum-structured matrices and the solution of Sylvester matrix equations.

We propose a recursive block diagonalization algorithm for computing bivariate functions of matrices of small to medium size, for which dense linear algebra is appropriate. The algorithm combines a blocking strategy, as in the Schur-Parlett scheme, and an evaluation procedure for the diagonal blocks. We discuss two implementations of the latter. The first is a natural choice based on Taylor expansions, whereas the second is derivative-free and relies on a multiprecision perturb-and-diagonalize approach. In particular, the appropriate use of multiprecision guarantees backward stability without affecting the efficiency in the generic case. This makes the second approach more robust.

The method has cubic complexity and it is closely related to the well-known Bartels-Stewart algorithm for Sylvester matrix equations when applied to $\frac{1}{x+y}$. We validate the performances of the proposed numerical method on several problems with different conditioning properties.

Joint work with S. Massei

References

- [1] D. KRESSNER, *Bivariate matrix functions*, Operators and Matrices 8, 449–466 (2014).