Graph Topological Stability via Matrix Differential Equations

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Understanding the underlying topology of data requires representing more than dyadic relationships between agents and poses the question of graph's generalizations that exploit higher-order interactions, [1]. Among the direct applications one can find neurology, chemistry, regulatory networks, PageRank, etc. One possible approach is the use of simplicial complexes, with simplices related through the boundary operators. Such operators satisfy the Hodge theory [2], and, thus, comprise higher-order Hodge Laplacians whose kernels correspond to different topological features in the graph. For example, 0-order Laplacians describe the connected components and 1-order Hodge Laplacians the 1-dimensional holes.

In the current work we discuss the topological stability of the graph through a spectral matrix nearness problem for the 1-order Hodge Laplacian. Specifically, the objective is to find the "smallest" perturbation of the graph's weights such that the number of 1-dimensional holes is increased at least by 1. Firstly, the work formulates the proper weighted generalization of the Hodge Laplacian and, then, suitably extends the constrained gradient flow method [3]. Method's performance is illustrated on synthetic quasi-triangulation datasets and transportation networks.

Joint work with N. Guglielmi *and* F. Tudisco

References

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