Numerical solution of the graph p-Laplacian partial eigen-problem

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In the last years the graph p-Laplacian eigenvalue problem has become central in applications related to machine learning, such as for example spectral clustering and classification. Several numerical approaches have been proposed to compute the variational eigenpairs as critical points of the p-Rayleigh quotient. However, the approximation of the partial p eigenspectrum is still an open issue.

We propose a new approach at the numerical approximation of the eigenpairs of the graph p-Laplacian. The approach originates from the Dynamic Monge-Kantorovich reformulation of the L^1 optimal transport problem developed in [1] and makes use of the convergence results presented in [2]. We introduce a family of Transport Energy functionals and define corresponding gradient flows (along descending and ascending direction) forming a sequence of generalized weighted linear graph-Laplacians whose eigenpairs converge in some sense to the graph p-Laplacian eigenpairs. Numerical experiments on realistic test cases show the ability of the developed numerical schemes to identify eigenpairs of the graph p-Laplacian for general values of the parameter p, including the limit cases $p = 1$ and $p = \infty$. We highlight the geometrical properties of the numerical eigenpairs and compare them to available and new theoretical findings.

Joint work with P. Deidda, M. Putti

References

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