Computation of generalized matrix functions with rational Krylov methods

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Generalized matrix functions [4] are an extension of the notion of standard matrix functions to rectangular matrices, defined using the SVD instead of an eigenvalue decomposition. In this talk, we consider the computation of the action of a generalized matrix function on a vector and we present a class of algorithms based on rational Krylov methods [3]. These algorithms incorporate as a special case previous methods based on the Golub-Kahan bidiagonalization [1]. By exploiting the quasiseparable structure of the projected matrices, we show that the basis vectors can be updated using a short recurrence, which can be seen as a generalization to the rational case of the Golub-Kahan bidiagonalization. We also prove error bounds that relate the error of these methods to uniform rational approximation on an interval containing the singular values of the matrix. The effectiveness of the algorithms and the accuracy of the bounds is illustrated with numerical experiments.

Joint work with A.A. Casulli.

References

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