Matrices associated to two conservative discretizations of Riesz fractional operators and related multigrid solvers

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Due to the non-local behavior of fractional differential operators, Fractional Diffusion Equations (FDEs) have been proven useful to model anomalous diffusion phenomena appearing in several applicative fields, like plasma physics or imaging. In this talk, we focus on a two-dimensional conservative steady-state Riesz fractional diffusion problem. As is typical for problems in conservative form, we adopt a Finite Volume (FV) discretization approach. Precisely, we use both classical FVs and the so-called Finite Volume Elements (FVEs). In the FV method we use fractionally-shifted Grünwald formulas to discretize the Riemann-Liouville fractional derivatives at control volume faces in terms of function values at the nodes. In the FVE case, the solution is approximated in the space of C^0 finite elements and then fractionally derived using exact formulas for fractional derivatives of a polynomial. While FVEs have already been applied in the context of FDEs, classical FVs have only been applied in first order discretizations. By exploiting the Toeplitz-like structure of the resulting coefficient matrices, we perform a qualitative study of their spectrum and conditioning through their symbol, leading to the design of a second order FV discretization. This same information is leveraged to discuss parameter-free symbol-based multigrid methods for both discretizations. Tests on the approximation error and the performances of the considered solvers are given as well.

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