Solving sparse-dense least squares

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Our focus is on efficient solution of the unconstrained linear least squares (LS) problems that can be provided in the form

$$\min_x \|Ax - b\|_2, \tag{1}$$

where $A \in \mathcal{R}^{m\times n}$ $(m \geq n)$ is a large sparse matrix and $b \in \mathcal{R}^m$ is given. Solving the LS problems numerically is often significantly harder than solving large and sparse systems of linear equations. One important obstacle to get efficient solvers is internal sparsity structure of the normal equations. This structure does not need to be visible when applying black-box direct methods. But the structure may visibly influence solving large problems when using preconditioned iterations. Our presentation will discuss the LS problems in which the system matrix contains rows with very different densities.

There are several classical contributions to solving the LS problem that focus on the problem, and one can find them summarized in the monograph by Åke Björck and some later surveys. In this presentation, we discuss a number of solution approaches. For example, we consider direct sparse-dense preconditioning, Schur complement reductions, combination of the QR factorization with stretching [1] as well as the null-space approach [2]. Experimental problems demonstrate not only strengths but also limitations of various approaches to solve these sparse-dense LS problems.

Joint work with J. Scott

References

[1] J. A. Scott and M. Tůma, *A computational study of using black-box QR solvers for large-scale sparse-dense linear least squares problems*, ACM Transactions on Mathematical Software, 2021, to appear.

[2] J. A. Scott and M. Tůma, *A null-space approach for symmetric saddle point systems with a non zero (2,2) block*, preprint, 2021.