On linear algebra in interior point methods for solving ℓ_1-regularized optimization problems

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The efficiency and robustness of interior point methods depend on the availability of effective linear algebra solvers, able to take into account the features of the optimization problem to be solved [1]. Here we consider nonsmooth convex problems of the form

$$\min_{\mathbf{x}} f(\mathbf{x}) + \tau_1\|\mathbf{x}\|_1 + \tau_2\|L\mathbf{x}\|_1, \quad \text{s.t. } A\mathbf{x} = \mathbf{b},$$

where $f : \mathbb{R}^n \mapsto \mathbb{R}$ is twice continuously differentiable and convex, $L \in \mathbb{R}^{l\times n}$, $A \in \mathbb{R}^{m\times n}$, $\mathbf{b} \in \mathbb{R}^m$, $m \leq n$, and $\tau_1, \tau_2 > 0$. Problems of this form arise in several real-life applications, e.g. portfolio optimization, signal and image processing, machine learning, compressed sensing, and are usually solved by specialized versions of first-order methods. For these problems we develop variants of an Interior Point-Proximal Method of Multipliers that use proper linear algebra solvers and take advantage of the expected sparsity in the optimal solution. We show the effectiveness of our approach versus state-of-the-art first-order methods by focusing on the problem of restoring images corrupted by Poisson noise. Further details and applications are given in [2].

Joint work with V. De Simone, D. di Serafino, J. Gondzio, *and* S. Pougkakiotis

References

[1] M. D'Apuzzo, V. De Simone and D. di Serafino, *On mutual impact of numerical linear algebra and large-scale optimization with focus on interior point methods*, Computational Optimization and Applications, 45 (2010), pp. 283–310.

[2] V. De Simone, D. di Serafino, J. Gondzio, S. Pougkakiotis and M. Viola, *Sparse Approximations with Interior Point Methods*, to appear in SIAM Review (2021).