Analysis, Control and Inverse Problems for PDEs

November 26-30, 2018, Napoli

About the workshop

The aim of the workshop is to bring together researchers working on a wide range of subjects in applied Mathematics, such as Control and Stabilization of Partial Differential Equations (PDE), Optimal Control, Shape Optimization, Inverse Problems for PDEs, Numerical Analysis of Control Problems, in order to foster the opportunity to share recent results, techniques, ideas and projects related to the different areas represented among the participants.

The conference is framed in the context of the French-German-Italian Laboratoire International Associé (LIA), named COPDESC, on Applied Analysis and the four thematic areas of the LIA COPDESC will be represented.

The invited speakers are divided in:

- Plenary speakers (the plenary lectures are 40 minutes long)
- Senior invited speakers in two parallel sessions (the talks are 25 minutes long)
- Junior invited speakers in two parallel sessions (the junior talks are 15 minutes long) .

A poster session is scheduled on Wednesday 28th November 2018.

Organizing committee

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Control and indirect stabilization, examples and applications

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The control of PDE's arise in many applications and is studied since several decades. This talk will focus on the control and stabilization of coupled systems in cases for which only some of the equations are directly controlled or stabilized. This is called indirect control or indirect stabilization. We shall discuss some powerful methods to prove indirect controllability or indirect stabilization for linear coupled PDE's and give applications. We shall also present results for indirect stabilization of some finite dimensional nonlinear gradient systems for which continua of critical points exist.

Exponential Stability of Large BV Solutions in a Model of Granular flow

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We consider a 2×2 system of hyperbolic balance laws that describes the evolution of a granular material, with slow erosion and deposition, in terms of the thickness of a moving layer on top and of a standing layer at the bottom. The system is linearly degenerate along two straight lines in the phase plane and genuinely nonlinear in the subdomains confined by such lines. In particular, the characteristic speed of the first characteristic family is strictly increasing in the region above the line of linear degeneracy and strictly decreasing in the region below such a line. The non dissipative source term is the product of two quantities that are transported with the two different characteristic speeds.

The global existence of entropy weak solutions of the Cauchy problem for such a system was established by Amadori and Shen [1] for initial data with bounded but possibly large total variation, under the assumption that the initial height of the moving layer be sufficiently small.

In this paper we establish the Lipschitz $\mathbf{L^1}$-continuous dependence of the solutions on the initial data with a Lipschitz constant that grows exponentially in time. The proof of the $\mathbf{L^1}$-stability of solutions is based on the construction of a Lyapunov like functional equivalent to the $\mathbf{L^1}$-distance, in the same spirit of the functional introduced by Liu and Yang [3] and then developed by Bressan, Liu, Yang [2] for systems of conservation laws with genuinely nonlinear or linearly degenerate characteristic fields.

References

[1] D. Amadori and W. Shen, Global existence of large BV solutions in a model of granular flow, *Comm. Part. Diff. Equations* **34** (2009) 1003–1040.

[2] T.P. Liu, T. Yang, L^1-stability for 2×2 systems of hyperbolic conservation laws, *J. Amer. Math. Soc.*, **12** (1999), no. 3, pp. 729-774.

[3] A. Bressan, T.P. Liu, T. Yang, L^1 stability estimates for $n \times n$ conservation laws, *Arch. Rational Mech. Anal.*, **149**, 1-22 (1999).

Joint work with: Laura Caravenna (*Università di Padova*), Cleopatra Christoforou (*University of Cyprus*).

Control of water waves and quasilinear evolution PDEs

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The talk deals with recent results of exact controllability, with internal control, for dispersive quasilinear evolution PDEs in the space-periodic setting: the gravity-capillary water wave equations, and quasilinear perturbations of the Korteweg-de Vries (KdV) and nonlinear Schrödinger (NLS) equations.

Two different approaches are considered: the work [1] is based on microlocal analysis with a quasilinear scheme, while [2,3,4] rely on a new Nash-Moser theorem, which is sharp regarding a certain regularity issue.

References

[1] T. Alazard, P. Baldi and D. Han-Kwan, Control of water waves, *J. Eur. Math. Soc. (JEMS)*, **20** (2018), 657-745.

[2] P. Baldi, G. Floridia, E. Haus, Exact controllability for quasi-linear perturbations of KdV, *Anal. PDE* **10** (2017), 281-322.

[3] P. Baldi, E. Haus, A Nash-Moser-Hörmander implicit function theorem with applications to control and Cauchy problems for PDEs, *J. Funct. Anal.* **273** (2017), 3875-3900.

[4] P. Baldi, E. Haus, R. Montalto, Controllability of quasi-linear Hamiltonian NLS equations, *J. Differential Equations*, **264** (2018), 1786-1840.

Joint works with: Thomas Alazard (*École Normale Supérieure de Paris-Saclay*), Daniel Han-Kwan (*École Polytechnique, France*), Giuseppe Floridia (*Università di Napoli Federico II*), Emanuele Haus (*Università di Napoli Federico II*), Riccardo Montalto (*Università di Milano*).

On the regularity of abnormal minimizers for rank 2 sub-Riemannian structures

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We discuss the problem of regularity of length-minimizers in sub-Riemannian geometry. We prove the C^1 regularity for a class of abnormal length-minimizers in rank 2 sub-Riemannian structures. As a consequence of our result, all length-minimizers for rank 2 sub-Riemannian structures of step up to 4 are of class C^1.

References

[1] Davide Barilari, Yacine Chitour, Frédéric Jean, Dario Prandi, Mario Sigalotti, On the regularity of abnormal minimizers for rank 2 sub-Riemannian structures, *Preprint Arxiv*, (2018).

Joint work with: Yacine Chitour, Frédéric Jean, Dario Prandi, Mario Sigalotti

Some regularity results on double phase variational integrals

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I will describe some recent updates in the regularity theory for minimizers of non-autonomous functionals of the Calculus of Variations, so-called double phase functionals. I will try to show in this respect how a unified proof, simpler than all the previous ones, allowed us to catch borderline cases and to detect new, unexpected phenomena.

References

[1] P. Baroni, M. Colombo, G. Mingione, Regularity for general functionals with double phase, *Calc. Var. Partial Differential Equations*, **57 (2)** (2018).

Necessary conditions for infinite horizon optimal control problems under state constraints and Hamilton-Jacobi-Bellman equations

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In this talk I will discuss sufficient conditions for Lipschitz regularity of the value function for an infinite horizon optimal control problem subject to state constraints. I focus on problems with cost functional admitting a discount rate factor and allow time dependent dynamics and Lagrangian. Furthermore, state constraints may be unbounded and may have a nonsmooth boundary. Lipschitz regularity is recovered as a consequence of estimates on the distance of a given trajectory of control system from the set of all its viable (feasible) trajectories, provided the discount rate is sufficiently large (cfr. [2]). I will talk about first order necessary optimality conditions: a constrained maximum principle and sensitivity relations involving generalized gradients of the value function (cfr. [1]). Finally, I will address nonautonomous Hamilton-Jacobi-Bellman equations, with time-measurable data, and their weak solutions: an existence and uniqueness result for solutions to the H-J-B equation associated with an infinite horizon control problem is discussed (cfr. [3]).

References

[1] V. Basco, P. Cannarsa, and H. Frankowska, Necessary conditions for infinite horizon optimal control problems with state constraints, *Mathematical Control & Related Fields*, **8(3&4)** (2018), pp. 535-555

[2] V. Basco and H. Frankowska, Lipschitz continuity of the value function for the infinite horizon optimal control problem under state constraints, To appear.

[3] V. Basco and H. Frankowska, Hamilton-Jacobi-Bellman Equations with Time-Measurable Data and Infinite Horizon, Submitted for publication.

Joint work with: Piermarco Cannarsa (*University of Roma "Tor Vergata"*), Hélène Frankowska (*Sorbonne Université*).

Null controllability of hypoelliptic equations on the whole space

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We formulate necessary geometrical conditions, and sufficient geometrical conditions for the null controllability of some hypoelliptic equations on the whole space: Ornstein-Uhlenbeck equations and quadratic equations with zero singular space.

The proof of the positive controllability results relies on a variante of the Lebeau-Robbiano's method, with projections that may not be spectral projections. The lack of commutation between the projection and the evolution system is compensated by appropriate smoothing properties of the equation.

For Ornstein-Uhlenbeck equations, we use a spectral inequality for the Fourier transform proved by Kovrijkine. For quadratic equations, we prove a new spectral inequality for the Hermite functions, by adapting Kovrijkine's proof.

Natural norm stabilization of unstable numerical methods

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In many problems from different applications, a partial differential equation arises that takes the following form: find $u \in V$, (V a Hilbert space), such that for all $v \in V$, we have $a(u, v) = \langle f, v\rangle$ with f a given element of the dual space V' and a a positive semidefinite bilinear form. For instance, this is the case of the Stokes equation, and of the equations arising from the application the Discontinuous Galerkin method to an elliptic PDE. It is well known that when discretizing equations of such a form, instability problems can arise. The remedy is either to choose the discretization space carefully, which might be difficult or even practically unfeasible, or to utilize some stabilization technique, allowing one to transform the unstable discrete problem into a stable one by adding either suitable elements to the discretization space, or (and this is the case that we are going to consider), suitable consistent terms to the equation itself, which penalize some residual, usually in a mesh dependent norm. Estimates on the resulting methods rely on the existence of direct and/or inverse inequalities relating such norm to the norm of the dual space where the residual naturally lives. In this talk we discuss the possibility of designing computable dual norms, and of using them in the design of stabilization terms. To illustrate this idea, we will propose a DG method for solving the Poisson equation on polygonal meshes, in which the unknown in the polygonal elements, the fluxes and the unknown on the edges, are all, independently, approximated by polynomials of degree k. Well posedness is achieved using a suitable "minus one norm" stabilization term penalizing the discrepancy between actual fluxes of the solution in the element and the unknown that independently discretizes such fluxes.

References

[1] S. Bertoluzza, Algebraic representation of dual scalar products and stabilization of unstable numerical methods, preprint (2018).

[2] S. Bertoluzza and D. Prada, A polygonal discontinuous Galerkin method with minus one stabilization, preprint (2018).

Joint work with: Ilaria Perugia (*Faculty of Mathematica, University of Vienna*), Daniele Prada (*CNR - IMATI "Enrico Magenes"*)

The self-adjointness of the Laplacian and the underlying geometry

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In this talk I will discuss the essential self-adjontness of the Laplace operator in relation with the underlying geometry. I will consider 2 different situations: 2D almost-Riemannian manifolds and 3D contact sub-Riemannian manifolds without a point.

A 2D almost-Riemannian manifold M is a 2D generalized Riemannian manifold (Grushin-like) in which a local orthonormal frame is given by a pair of vector fields that can become parallel. For such a manifold, in the generic case, the singular set $\mathcal{Z}$ (where it is not Riemannian) is a 1D manifold. I will show that the Laplace-Beltrami operator is essentially self-adjoint in each connected components of $M \setminus \mathcal{Z}$ meaning that no information can flow through the singular set via the heat, the Schroedinger or the wave equation, even if geodesics can cross it. For the heat equation I will give an interpretation of this phenomenon in terms of random walks.

For a 3D-contact sub-Riemannian manifolds M that is complete as metric space (e.g. the Heisenberg group), I will show that the hypoelliptic Laplacian is essentially self-adjoint in $M \setminus \{q\}$ where q is a point. This is a curious fact since in $\mathbf{R}^n \setminus \{q\}$ one has self-adjointness of the Laplacian iff $n \geq 4$. In a sense for the self-adjointness of the Laplacian, a 3D contact sub-Riemannian manifold behaves like a 4 dimensional space.

References

[1] A. Agrachev, D. Barilari, U. Boscain, *A Comprehensive Introduction to sub-Riemannian Geometry*, Cambridge University Press, , to appear.

[2] U. Boscain, C. Laurent, The Laplace-Beltrami operator in almost-Riemannian geometry. *Ann. Inst. Fourier* **63** (2013), no. 5, pp. 1739?1770.

[3] R. Adami, U. Boscain, V. Franceschi, D. Prandi, On the self-adjointness of 3D pointed sub-Laplacians. Preprint.

Joint work with: Riccardo Adami *Politecnico di Torino*, Valentina Franceschi *Orsay*, Camille Laurent *CNRS*, Dario Prandi *CNRS*.

A sharp estimate for Neumann eigenvalues of the Laplace-Beltrami operator for domains in a hemisphere

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We prove an isoperimetric inequality for the harmonic mean of the first $N-1$ non-trivial Neumann eigenvalues of the Laplace-Beltrami operator for domains contained in a hemisphere of $\mathbb{S}^N$.

Joint work with: Francesco Chiacchio (*Università degli Studi di Napoli Federico II, Italy*), Rafael D. Benguria (*P. Universidad Católica de Chile, Santiago, Chile*)

On Riccati equations arising in the optimal boundary control of certain PDE systems with predominant or full hyperbolic character

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In this talk we will discuss research results regarding Riccati equations which arise in the optimal control problem with quadratic functionals of i) a specific hyperbolic Partial Differential Equation (PDE) and ii) a class of systems of coupled hyperbolic-parabolic PDE's that display a 'predominant' hyperbolic character (in a sense to be rendered more precise). With regard to ii), it will be first recalled the theoretical framework of the LQ-problem for a class of systems with unbounded (dynamics and) control operators, introduced jointly with Paolo Acquistapace and Irena Lasiecka in 2005. Initially inspired by thermoelastic equations, the said framework proved over the years to be effective in treating PDE systems which model diverse, relevant, physical interactions (*viz.* mechanical-thermal, acoustic-structure, solid-fluid), yet allowing rather general (i.e. non-smoothing) observations in the functionals to be minimized. (In the analysis of either control problem, a crucial role is played by suitable model-specific "hidden" boundary regularity results that need to be established.) The Riccati theory developed for both the finite and infinite time horizon problems is now complete with the proof of uniqueness of their solutions.

As for i), we will discuss – necessarily, only briefly – an optimization problem for the Stokes-Moore-Gibson-Thompson equation, that is a linearization of a PDE model for the propagation of ultrasound waves. The feedback synthesis of an optimal control via the solution to a non-standard operator Riccati equation has been obtained, overcoming the challenges stemming from the hyperbolic dynamics subject to boundary control.

(The talk is based partly on joint work with Paolo Acquistapace (Pisa, Italy) and partly on joint work with Irena Lasiecka (University of Memphis, USA).)

Joint work with: Paolo Acquistapace (*Università di Pisa, ITALY*), Irena Lasiecka (*University of Memphis, USA*

Maximization of Neumann eigenvalues

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In this talk I will discuss the question of the maximization of the k-th eigenvalue of the Neumann-Laplacian among open sets with fixed volume. After an introduction to the topic and discussion about the existence of optimal geometries, I will focus on the low eigenvalues. The first non-trivial one is maximized by the ball, the result being due to Szego and Weinberger in the fifties. Concerning the second non-trivial eigenvalue, Girouard, Nadirashvili and Polterovich proved that the supremum in the family of planar simply connected domains of R^2 is attained by the union of two disjoint, equal discs. I will show that a similar statement holds in any dimension and without topological restrictions. In particular, this implies that the Polya conjecture for the Neumann eigenvalues holds for the second eigenvalue and for arbitrary domains. Some numerical computations based on relaxation on densities are also presented.

References

[1] D. Bucur and A. Henrot, Maximization of the second non-trivial Neumann eigenvalue, *Preprint CVGMT*, (2018)

Joint work with: Antoine Henrot (*Université de Lorraine*), Edouard Oudet (*Université Grenoble Alpes*)

Optimal reinforcing networks for elastic structures

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We study the optimal reinforcement of an elastic membrane, fixed at its boundary, by means of a connected one-dimensional structure. The problem consists in finding the optimal configuration for the stiffeners, the problem is then a shape optimization problem, where the admissible competing shapes are one-dimensional networks of prescribed length. We show the existence of an optimal solution that may present multiplicities, that is regions where the optimal structure overlaps. The case where the connectedness assumption is removed is also presented. Some numerical simulations are shown to confirm the overlapping phenomenon and to illustrate the complexity of the optimal structures when their total length becomes large.

References

[1] G. Alberti, G. Buttazzo, S. Guarino Lo Bianco, E. Oudet: Optimal one-dimensional reinforcement for elastic membranes. *Preprint, available at* http://cvgmt.sns.it

[2] G. Buttazzo, E. Oudet, B. Velichkov: A free boundary problem arising in PDE optimization. *Calc. Var.*, **54** *(2015), 3829–3856*

On the optimization of conservation laws at a junction

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I will present a traffic flow model at a junction where the dynamics follows the classical conservation law formulation introduced by Lighthill, Whitham and Richards. The usual approach to achieve uniqueness of solutions is based on a pointwise maximization criteria involving the solutions of boundary Riemann problems. I will adopt a different perspective following a control theoretic approach. Namely, I will consider a general optimization problem where the boundary data and the distributional parameters at the junction are regarded as controls. The goal is to select, on a given time interval $(0, T)$, (possibly non unique) solutions which maximize a suitable functional of the flux traces of the incoming edges (as maps over the whole interval $(0, T)$), among all entropy admissible solutions that preserve the conservation of cars through the junction and satisfy some distributional rules.

Joint work with: Fabio Ancona (*Università di Padova*), Giuseppe Maria Coclite (*Politecnico di Bari*), Mauro Garavello (*Università di Milano Bicocca*)

Conservation laws for a network of supply chains with discontinuous speed and finite buffer

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We introduce a new model for supply chains on a network based on conservation laws with discontinuous flux evolving on each arc (sub-chain) and on buffers of limited capacity in every junction (separating sub-chains). The dynamics of every arc is governed by a continuity equation describing the evolution of the density of objects processed by the supply chain. The flux is discontinuous at the maximal density since it admits different values according with the free or congested status of the supply chain. We provide a definition of viscosity solution on each arc for the corresponding (discontinuous) Hamilton-Jacobi equation and we show that the space derivative of such solutions are entropy weak solutions of the conservation law that keep trace of the (free-congested) status transition in the point of discontinuity of the flux. We then establish existence and uniqueness of solutions of the Hamilton-Jacobi equations on each arc and show that the map associating to a given buffer-queue the corresponding solution at the junction and then the resulting buffer-queue is a contractive transformation. This yields the well-posedness of the Cauchy problem at the junction with bounded and integrable initial data.

Joint work with: Fabio Ancona (*University of Padova*)

Bouligand–Landweber iteration for a non-smooth ill-posed problem

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This talk is concerned with the iterative regularization of a non-smooth nonlinear ill-posed problem where the forward mapping is merely directionally but not Gâteaux differentiable. Using the Bouligand derivative of the forward mapping, a Landweber-type iteration is derived that converges strongly for exact data as well as in the limit of vanishing data if the iteration is stopped according to the discrepancy principle. The analysis is based on the asymptotic stability of the proposed iteration, which is shown to hold under a generalized tangential cone condition. This is verified for an inverse source problem with a non-smooth Lipschitz continuous nonlinearity. Numerical examples illustrate the convergence of the iterative method.

References

[1] Christian Clason and Vu Huu Nhu, Bouligand–Landweber iteration for a non-smooth ill-posed problem, preprint (2018), arXiv:1803.02290

Joint work with: Vu Huu Nhu (*University Duisburg-Essen*)

Nonlinear Peridynamic Models

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Some materials may naturally form discontinuities such as cracks as a result of scale effects and long range interactions. Peridynamic models such behavior introducing a new nonlocal framework for the basic equations of continuum mechanics. In this lecture we consider a nonlinear peridynamic model and discuss its well-posedness in suitable fractional Sobolev spaces.

Joint work with: Serena Dipierro (Università di Milano), Francesco Maddalena (Politecnico di Bari), and Enrico Valdinoci (Università di Milano).

From the water clocks to the regulation of rivers

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A control system is a dynamical system on which one can act thanks to what is called the control. For example, in a car, one can turn the steering wheel, press the accelerator pedal etc. These are the control(s). The are at least two fundamental problems in control theory. The first problem is the controllability problem. One starts from a given situation and there is a given target. The controllability problem is to see if, by using some suitable controls depending on time, one can move from the given situation to the prescribed target. If this is possible, one can also try to do it and minimize some cost. The second problem is the stabilization problem, which is also very important for numerous applications. One can understand it with the classical experiment of an upturned broomstick on the tip of one's finger: In order to avoid the fall down, one moves the finger in a suitable way in order to stabilize this unstable equilibrium. In this talk key historical steps are presented. This includes the Ctesibius water clock, the Watt regulator, the Farcot servo-motor, the Sperry autopilot, the seminal Maxwell 1868 article "On governors" and the Lyapunov thesis. Then more recent results linking controllability and stabilization are detailed. Finally we present methods to stabilize various control systems. A special emphasis will be put on the regulation of rivers.

Simultaneous determination of the drift and diffusion coefficients in stochastic differential equations

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In this talk, we consider a one-dimensional Itô diffusion process X_t with possibly nonlinear drift and diffusion coefficients. I will show that, when the diffusion coefficient is known, the drift coefficient is uniquely determined by an observation of the expectation of the process during a small time interval, and starting from values X_0 in a given subset of $\mathbb{R}$. With the same type of observation, and given the drift coefficient, I also show that the diffusion coefficient is uniquely determined. When both coefficients are unknown, they are simultaneously uniquely determined by the observation of the expectation and variance of the process, during a small time interval, and starting again from values X_0 in a given subset of $\mathbb{R}$.

Reference

[1] M. Cristofol and L. Roques, Simultaneous determination of the drift and diffusion coefficients in stochastic differential equations, *Inverse problems*, **33.** (2017), 095006 (12pp).

Joint work with: L. Roques (*INRA Avignon France*),

Estimation of flow geometry and wall shear stress from magnetic resonance measurements

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Magnetic resonance velocimetry allows to obtain high resolution images of density and velocity fields in human arteries and other non-accessible vessels. Typical quantities of interest in such applications, like wall shear stress or pressure drop, are not directly observable, but have to be determined from these data by some sort of post-processing. In this talk, we investigate the stable identification of flow geometry and wall shear stress from such magnetic resonance images. We first discuss the ill-posedness of the geometry identification and the sub-sequent wall shear stress estimation and therby explain the observations made by other researchers. Based on these considerations, we propose a three step strategy for the stable recovery of both quantities of interest relying on variational regularization techniques whose only realization requires the numerical solution of certain partial differential equations. In addition, we discuss the possible improvement of the flow reconstruction by fluid dynamic consistent filtering, which consists of fitting a problem adapted fluid dynamic model to the flow measurements by an optimal control approach.

References

[1] H. Egger, T. Seitz and C. Tropea, Enhancement of flow measurements using fluid-dynamic constraintsTitle of the paper, *J. Comput. Phys.*, **344** (2017), pp. 558-574

[2] H. Egger, A. Krafft, N. Shokina, and G. Teschner, Estimation of flow geometry and wall shear stress from magnetic resonance imaging data, TU Darmstadt, 2018.

Joint work with: T. Seitz, G. Teschner, C. Tropea (*TU Darmstadt*), A. Krafft, N. Shokina (*University Medical Center Freiburg*)

Minimal time issues for the observability of Grushin like equations

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The goal of this talk is to discuss the observability properties of degenerate parabolic equations. Namely, when considering a Grushin operator of the form $\partial_t - \partial_{xx} - |x|^2\partial_{yy}$, there is a whole line of degeneracy at $x = 0$, which is known from recent works to strongly modify the observability properties of this operator. In particular, while the usual heat equation is observable from any open set in any small time (we will recall the precise definition of observability we are dealing with), the Grushin operator may require some strictly positive minimal time to be observable, depending on the observation set. The question thus is to give explicit estimates on this minimal time and to link it with the geometry under consideration. We shall present several results in this direction, in which we are able to characterize completely the minimal time required for observability. In order to do this, we will use

Carleman estimates to estimate precisely the cost of observability of the family of 1-d heat equations whose operators are $\partial_t - \partial_{xx} + n^2|x|^2$ in the asymptotics $n \to \infty$.
This is a joint work with Karine Beauchard (ENS Rennes) and Jérémi Dardé (Institut de Mathématiques de Toulouse).

References

[1] K. Beauchard, J. Dardé, and S. Ervedoza. *Minimal time issues for the observability of Grushin-type equations.* https://hal.archives-ouvertes.fr/hal-01677037, Jan. 2018.

Joint work with: Karine Beauchard (*ENS Rennes*), Jérémi Dardé (*Institut de Mathématiques de Toulouse*).

A degenerate population equation: Carleman estimates and null controllability

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We present a degenerate model describing the dynamics of a population depending on time, age and space. We assume that the degeneracy occurs at the space boundary and we focus on null controllability reults via Carleman estimates for the associated adjoint problem.

Invariance of Sets under Semilinear Systems

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Many models of applied sciences involve dynamical systems under state constraints, that is restrictions on some of variables are imposed by the model. As for instance in models of sustainable development, some populations have to be kept over a prescribed level. In this talk I will discuss recent results on invariance and viability of semilinear systems.

Asymptotic stability of the gradient flow of nonlocal energies

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I will discuss short time existence and long time stability of a class of equations modeling the evolution of the interface between an elastic material and a material void, controlled by mass diffusion within the surface. These equations appear as the H^{-1}- gradient flow of an energy given by the sum of the area of the interface plus a non local volume term. Our stability results are new even in the simplest case of the surface diffusion equation

$$V_t = \Delta_{\Gamma_t} H_t,$$

where V_t is the normal velocity at the boundary Γ_t of the evolving set, H_t is the mean curvature of Γ_t and $\Delta_{\Gamma t}$ is the Laplace-Beltrami operator on Γ_t.

More in general, we shall consider the following evolution equation

$$V_t = \Delta_{\Gamma_t}\big(H_t - W(E(u_t))\big),$$

where $F_t \subset\subset \Omega$, Ω is a given open set in $\mathbb{R}^3$, $u_t : \Omega \setminus F_t \mapsto \mathbb{R}^3$ is the solution at time $t > 0$ of the minimum problem

$$\min\left\{ \int_{\Omega\setminus F_t} W(E(v))\,dx :\ v \in H^{1,2}(\Omega \setminus F_t; \mathbb{R}^3),\ u = u_o \text{ on } \partial\Omega \right\},$$

$E(v) = \big(\nabla v + \nabla^T v\big)/2$ is the symmetric part of ∇v and $W(A)$ is a strictly positive definite quadratic form acting on 3×3 symmetric matrices A.

The results presented in this talk are contained in [1], [2].

References

[1] E. Acerbi, N. Fusco, V. Julin, M. Morini, Nonlinear stability results for the modified Mullins-Sekerka and the surface diffusion flow, preprint (2017) to appear on *J. Diff. Geo.*, http://cvgmt.sns.it/paper/3087/

[2] N. Fusco, V. Julin, M. Morini, The surface diffusion flow with elasticity in three dimensions, preprint (2018), http://cvgmt.sns.it/paper/4082/

One-side boundary controllability of the p-system

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We consider the p-system on an interval, in the context of weak entropy solutions. On one side of the interval is imposed a boundary condition (for instance the null velocity), while on the other side of the interval, the boundary condition is left as a control, that is, as a parameter that one can choose to influence the system. We prove a result of controllability toward constants states, namely, that it it possible, starting from a small initial state in BV, to reach any constant state compatible with the boundary conditions. This is in sharp contrast with some other 2×2 strictly hyperbolic systems with genuinely nonlinear characteristics fields for which a result by Bressan and Coclite shows that this is not possible, even when controlling on both sides.

References

[1] Alberto Bressan and Giueseppe Maria Coclite. On the boundary control of systems of conservation laws. *SIAM J. Control Optim.* **41** no. 2 (2002), pp. 607-622.

Joint work with: Fabio Ancona (*Università di Padova*) and Tien Khai Nguyen (*North Carolina State University*).

Traffic control by autonomous vehicles

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We present a coupled PDE-ODE model describing the interaction of a controlled vehicle with the surrounding traffic. The traffic flow is described by the classical LWR macroscopic model, while the velocity of the controlled vehicle, which acts as a control, adapts to the downstream traffic conditions. At the same time, the presence of the vehicle induces a moving bottleneck, hindering traffic flow. We prove the well-posedness of solutions via a wave-front tracking algorithm [2, 3, 4]. We also present a finite volume scheme to compute approximate solutions [1]. We then apply a MPC (Model Predictive Control) algorithm to optimize the traffic flow in terms of fuel consumption minimization, showing that this also reduces the Average Travel Time and the queue length upstream a fixed bottleneck [5].

References

[1] C. Chalons, M. L. Delle Monache, and P. Goatin. A conservative scheme for non-classical solutions to a strongly coupled PDE-ODE problem. *Interfaces Free Bound.*, 19(4):553–570, 2017.

[2] M. L. Delle Monache and P. Goatin. Scalar conservation laws with moving constraints arising in traffic flow modeling: an existence result. *J. Differential Equations*, 257(11):4015–4029, 2014.

[3] M. Garavello, P. Goatin, T. Liard and B. Piccoli. Traffic control by autonomous vehicles. In preparation.

[4] T. Liard and B. Piccoli. Well-posedness for scalar conservation laws with moving flux constraints. *ArXiv e-prints*, Jan. 2018.

[5] G. Piacentini, P. Goatin, and A. Ferrara. Traffic control via moving bottleneck of coordinated vehicles. Proceedings of the 15th IFAC Symposium on Control in Transportation Systems, to appear.

Joint works with: Christophe Chalons (*Université de Versailles*), Maria Laura Delle Monache (*Inria Rhone-Alpes*), Antonella Ferrara (*Università di Pavia*), Mauro Garavello (*Università di Milano - Bicocca*), Thibault Liard (*Inria Rhone-Alpes*), Giulia Piacentini (*Università di Pavia*), Benedetto Piccoli (*Rutgers University - Camden*)

Hybrid System Theory for Gas Network Operation

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We use the framework of hybrid dynamical systems to model, simulate and optimize transient operation of gas networks including discrete desicisions on valve positions and compressor routes. We show that the problem can be posed as switching among a number of abstract semilinear evolutions on a Hilbert space. Moreover, we derive gradient representations for switching times and mode insertions using suitable adjoint problems in order to apply gradient descent methods for computing optimal switching sequences in some local first order sense. We apply these techniques to decompose a gas network into regions of transient and stationary behavior that balances numerical costs and accuracy of the solution. The performance is demonstrated on numerical examples.

References

[1] F. Rüffler and F.M. Hante, Optimal Switching for Hybrid Semilinear Evolutions, *Nonlinear Analysis: Hybrid Systems*, Vol. 22 (2016), pp. 215–227, DOI:10.1016/j.nahs.2016.05.001.

[2] F. Rüffler, V. Mehrmann and F.M. Hante, Optimal Model Switching for Gas Flow in Pipe Networks, *Networks and Heterogeneous Media*, Vol. 13 (2018), DOI:10.3934/nhm.2018029.

Stabilization of 1D nonlinear hyperbolic equations in fluid mechanics

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In this talk, we will discuss the stabilization of 1D nonlinear hyperbolic equations in fluid mechanics. We will first see that the Saint-Venant equations, a well-known model for shallow waters used in practice for the regulation of navigable rivers, and the isentropic Euler equations have a particular structure that enables the stabilization of any of their regular steady-states by simple boundary controls, whatever the source term is. This is true even if the physical data associated (slope, friction, etc.) are unknown. This feat comes from the existence of a remarkable local entropy that we will discuss. Then, we will see how to do when the steady-state is not regular anymore but includes a shock. This method will be conducted on Burgers and Saint-Venant equations.

References

[1] A. Hayat and P. Shang, A quadratic Lyapunov function for Saint-Venant equations with arbitrary friction and space-varying slope , preprint (2017) to appear on *Automatica*.

[2] G. Bastin, J-M. Coron, A. Hayat, P. Shang, Boundary feedback stabilization of hydraulic jumps, preprint (2018)

[3] G. Bastin, J-M. Coron, A. Hayat, P. Shang, Exponential boundary feedback stabilization of a shock steady state for the inviscid Burgers equation, preprint (2017)

Joint work with: Georges Bastin (*Université catholique de Louvain*), Jean-Michel Coron (*Sorbonne Université*) and Peipei Shang (*Tongji University*)

A DeGiorgi type conjecture for minimal solutions to a nonlinear Stokes equation

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The aim is to study the symmetry of transition layers in Ginzburg-Landau type functionals for divergence-free maps in $\mathbb{R}^N$. Namely, we determine a class of nonlinear potentials such that the minimal transition layers are one-dimensional. In particular, this class includes in dimension $N = 2$ the nonlinearities w^2 with w being an harmonic function or a solution to the wave equation, while in dimension $N > 2$, this class contains a perturbation of the standard Ginzburg-Landau potential as well as potentials having $N + 1$ wells with prescribed transition cost between the wells. For that, we develop a theory of calibrations for divergence-free maps in $\mathbb{R}^N$ (similar to the theory of entropies for the Aviles-Giga model when $N = 2$).

References

[1] R. Ignat and A. Monteil, A DeGiorgi type conjecture for minimal solutions to a nonlinear Stokes equation, *arXiv:1804.07502*.

Joint work with: Antonin Monteil (*Louvain-la-Neuve, Belgium*)

Concentration analysis of brittle damage

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This talk is concerned with an asymptotic analysis of a variational model of brittle damage, when the damaged zone concentrates into a set of zero Lebesgue measure and, at the same time, the stiffness of the damaged material becomes arbitrarily small. In a particular non-trivial regime, concentration leads to a limit energy with linear growth as typically encountered in plasticity. I will show that, while the singular part of the limit energy can be easily described, the identification of the bulk part of the limit energy requires a subtler analysis of the concentration properties of the displacements.

Joint work with: Jean-François Babadjian (*Université Paris-Sud*), Filip Rindler (*University of Warwick*)

On continuity of solutions for parabolic control systems and input-to-state stability

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We study minimal conditions under which mild solutions of linear evolutionary control systems are continuous for arbitrary bounded input functions. This question naturally appears when working with boundary controlled, linear partial differential equations. Here, we focus on parabolic equations which allow for operator-theoretic methods such as the holomorphic functional calculus. Moreover, we investigate stronger conditions than continuity leading to input-to-state stability with respect to Orlicz spaces. This also implies that the notions of input-to-state stability and integral-input-to-state stability coincide if additionally the uncontrolled equation is dissipative and the input space is finite-dimensional.

An Isoperimetric Problem with a Nonlocal Term

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In this talk I will present some results concerning the problem

$$\min_{\Omega\subset\mathbb{R}^n}\left\{P(\Omega)+\int_\Omega\int_\Omega\frac{1}{|x-y|^{n-2}}dxdy-K\int_\Omega\frac{1}{|x|^{n-2}}dx:\ |\Omega|=m\right\}.$$

where $K\geq 0$. I will first discuss about the issue of nonexistence of minimizers in the three dimensional case when m is too large and then I will talk about some minimality properties of balls centered at the origin.

References

[1] L., An isoperimetric problem with a Coulombic repulsion and attractive term, *ESAIM: COCV*, published online (2018)

[2] L., A short proof of a nonexistence result (2018), preprint, http://cvgmt.sns.it/paper/3932/

Maximum principles for some elliptic systems

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Consider the quasilinear system

$$- div\{a[x, u(x)]Du(x)\} = 0 \tag{1}$$

where $u : \Omega \subset \mathbb{R}^n \to \mathbb{R}^N$ and $a_{ij}^{\alpha\beta}[x, y]$ are measurable with respect to x, continuous with respect to y, bounded and elliptic. If we write explicitly (1) we get

$$\begin{cases} -\sum\limits_{i=1}^{n} D_i \left(\sum\limits_{j=1}^{n} a_{ij}^{11} D_j u^1 + \sum\limits_{j=1}^{n} a_{ij}^{12} D_j u^2 + ... + \sum\limits_{j=1}^{n} a_{ij}^{1N} D_j u^N \right) = 0 \\ -\sum\limits_{i=1}^{n} D_i \left(\sum\limits_{j=1}^{n} a_{ij}^{21} D_j u^1 + \sum\limits_{j=1}^{n} a_{ij}^{22} D_j u^2 + ... + \sum\limits_{j=1}^{n} a_{ij}^{2N} D_j u^N \right) = 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ -\sum\limits_{i=1}^{n} D_i \left(\sum\limits_{j=1}^{n} a_{ij}^{N1} D_j u^1 + \sum\limits_{j=1}^{n} a_{ij}^{N2} D_j u^2 + ... + \sum\limits_{j=1}^{n} a_{ij}^{NN} D_j u^N \right) = 0 \end{cases}$$

that is a system of N equations where $u = (u^1, u^2, ..., u^N)$. In the case $N = 1$, we have one equation and we get maximum principle. In the case of many equations, De Giorgi's counterexample shows that, in general, we cannot have a maximum modulus principle, see [1]. So the effort is finding additional assumptions on coefficients $a_{ij}^{\alpha\beta}[x, y]$ that keep away De Giorgi's counterexample and allow for maximum principle. We will present the results contained in [2].

References

[1] De Giorgi E., Un esempio di estremali discontinue per un problema variazionale di tipo ellittico, *Boll. Un. Mat. Ital.*, **1** (1968), pp. 135-137

[2] Leonardi S. - Leonetti F. - Pignotti C. - Rocha E. - Staicu V., Maximum principles for some quasilinear elliptic systems, preprint (2018), to appear on *Nonlinear Anal.*

Joint work with: Salvatore Leonardi (*University of Catania*), Cristina Pignotti (*University of L'Aquila*), Eugenio Rocha (*University of Aveiro*), Vasile Staicu (*University of Aveiro*).

Internal controls for a problem with fractional Laplacian using finite-difference method

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We consider a finite-difference semi-discrete scheme for the approximation of internal controls of a one-dimensional evolution problem of hyperbolic type involving the spectral fractional Laplacian. The continuous problems is controllable in arbitrary small time. However, the high frequency numerical spurious oscillations lead to a loss of the uniform (with respect to the mesh-size) controllability property of the semi-discrete model in the natural setting. For all initial data in a natural energy space, if we filter the high frequencies of these initial data in an optimal way, we restore the uniform controllability property in arbitrary small time. The proof is mainly based on the moment method.

References

[1] Pierre Lissy and Ionel Roventa, Optimal approximation of internal controls for a wave-type problem with fractional Laplacian using finite-difference method, *Submitted*, (2018), pp. 33.

Joint work with: Ionel Roventa, (*Department of Mathematics, University of Craiova*).

Quadratic Controllability

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In the work [1], we investigate the controllability of nonlinear scalar-input parabolic systems. When the linearized system is not controllable, we introduce a systematic method to study the second-order approximation of the system. We exhibit systems for which this method either leads to negative or positive results, leading to a panorama of possible behaviors.

Our systems are set on the line segment $x \in (0, \pi)$ and take the form

$$\begin{cases} \partial_t z(t,x) - \partial_{xx} z(t,x) = u(t)\Gamma[z(t)](x), \\ \partial_x z(t,0) = \partial_x z(t,\pi) = 0, \\ z(0,x) = z_0(x), \end{cases} \tag{2}$$

where $z_0 \in L^2(0,\pi)$ is the initial data, $u \in L^\infty(0,T)$ is a control to be chosen and $\Gamma : H^1_N(0,\pi) \to H^{-1}_N(0,\pi)$ is a nonlinearity. When the linearized system misses some directions, we prove that the controllability or non-controllability of the system can sometimes be decided by looking at the second-order expansion. We recover phenomenons known in finite dimensions (see [2]) and we exhibit new behaviors specific to the infinite dimensional setting.

References

[1] Karine Beauchard and Frédéric Marbach, Unexpected quadratic behaviors for the small-time local null controllability of scalar-input parabolic equations, preprint (2017), `arXiv:1712.09790`

[2] Karine Beauchard and Frédéric Marbach, Quadratic obstructions to small-time local controllability for scalar-input systems, *J. Differential Equations*, **264(5)** (2018), pp. 3704-3774, `doi:10.1016/j.jde.2017.11.028`

Joint work with: Karine Beauchard (*Univ Rennes, CNRS, IRMAR - UMR 6625, F-35000 Rennes, France*)

QHD: when fluid dynamics meets with quantum mechanics

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The talk will be organized to present the following items:

a) Crash introduction to the the problem and the physics behind it

b) Very short overview of the theory for finite energy weak solutions by P.Antonelli and P.M [1],[2], with the nonuniqueness results, via convex integration, by D.Donatelli, E.Feireisl and P.M. [3]

c) A Schrödinger free theory in 1-D, where we assume finite energy, finite mass and finite L^2 norm of the chemical potential. By P.Antonelli, P.M. and H.Zheng. [4]

d) Stability of infinite energy solution in 1-D via a complex "Cole - Hopf" type transform [5]

e) Some additional stability results (linearization and application of the results of S. Gustafson, K. Nakanishi and T.-P. Tsai)

References

[1] P. Antonelli, P. Marcati, On the finite energy weak solutions to a system in Quantum Fluid Dynamics, no. 2, *Comm. Math. Phys.*, **287** (2009), pp. 657-686

[2] P. Antonelli, P. Marcati, The Quantum Hydrodynamics system in two space dimensions. *Arch. Rat. Mech. Anal.* **203** (2012), pp. 499-527.

[3] D. Donatelli, E. Feireisl and P. Marcati, Well/ill posedness for the Euler- Korteweg-Poisson system and related problems. *Communications in Partial Differential Equations*, **40** (2015), pp.1314-1335.

[4] Global strong solutions to 1-D Quantum Hydrodynamics with the chemical potential in L^2. *Submitted*

[5] P.Antonelli, P.M, H.Zheng Stability for the Quadratic Derivative Nonlinear Schrödinger Equation and applications to the Korteweg–Kirchoff type Euler equations for Quantum Hydrodynamics. *Submitted*

Joint results with: Paolo Antonelli, (*GSSI*), Hao Zheng (*GSSI*)

A Bolza problem in Wasserstein space

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A multi-agent system is a system in the finite-dimensional Euclidean space where the number of possibly interacting agents is so large that only a statistical description of the state of the system is actually available. A common way to model such kind of systems is to describe the state of the system at time t by mean of a Borel measure μ_t where, for each Borel subset A of $\mathbb{R}^d$, the quotient $\mu_t(A)/\mu_t(\mathbb{R}^d)$ represents the fraction of the total number of agents that are present in the set A at time t over the total number of agents. In the case where neither creation nor destruction of agents are allowed, we normalize the total mass to the constant 1, thus μ_t becomes a time-depending probability measure. We consider such a system subject to a centralized controller aiming to minimize a cost function of Bolza type. We formulate the minimization problem as a problem for a dynamics in the Wasserstein space represented by a controlled continuity equation describing the macroscopical evolution of the system. We prove that the value function V of the problem solves a Hamilton-Jacobi equation in the Wasserstein space in a suitable viscosity sense, and prove a comparison principle for such an equation, thus characterizing V as the unique viscosity solution of the Hamilton-Jacobi equation associated to the problem.

References

1. G. Cavagnari and A. Marigonda: Attainability Property For A Probabilistic Target In Wasserstein Spaces. *preprint 2018.*
2. C. Jimenez, A. Marigonda and M. Quincampoix: Optimal Control of Multiagent Systems in Wasserstein Space. *preprint 2018.*
3. G. Cavagnari, A. Marigonda and B. Piccoli: Averaged time-optimal control problem in the space of positive Borel measures, ESAIM: COCV, Volume 24, Number 2, April–June 2018, pp. 721 - 740
4. A. Marigonda and M. Quincampoix: Mayer Control Problem with Probabilistic Uncertainty on Initial Positions and Velocities. *J. Diff. Eq.* 264(5):3212 – 3252, 2018.
5. G. Cavagnari, A. Marigonda, K. T. Nguyen and F. S. Priuli: Generalized control systems in the space of probability measures. Set-Valued and Variational Analysis, vol. 25, n. 2, pp.1-29 (2017).
6. G. Cavagnari, A. Marigonda and B. Piccoli: Optimal synchronization problem for a multi-agent system. Networks and Heterogeneous Media (NHM), vol. 12, n. 2, pp. 277-295 (2017).
7. G. Cavagnari: Regularity results for a time-optimal control problem in the space of probability measures. Mathematical Control and Related Fields (MCRF), vol. 7, n. 2, pp. 213-233 (2017).

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Explicit Bayesian inversion with adaptive low-rank approximation

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In recent years, the statistical Bayesian approach became a popular tool to alleviate the inherent ill-posedness of inverse problems by assigning probability densities to the considered calibration parameters. This stochastic regularisation method renders the numerical treatment of an inverse problem governed by a PDE as a challenging but relevant task. Based on recent advances in the adaptive handling of high-dimensional parametric partial differential equations, we employ low-rank techniques to present a sampling-free approach with an explicit representation of the parameter densities.

We discuss the advantages of tensor decomposition, which are employed for the adaptive evaluation of the random PDE in orthogonal chaos polynomials and the subsequent high-dimensional quadrature of the log-likelihood. All required computations can then be carried out efficiently in the low-rank format and discretization parameters are adjusted adaptively based on a posteriori error estimators or indicators.

Numerical experiments, involving diffusion and scattering as examples of a more general framework, demonstrate the performance and confirm the theoretical results.

References

[1] Martin Eigel, Manuel Marschall and Reinhold Schneider, Sampling-free Bayesian inversion with adaptive hierarchical tensor representations, *Inverse Problems*, **Volume 34 no. 3** (2018)

[2] Martin Eigel, Manuel Marschall, Max Pfeffer and Reinhold Schneider, Adaptive stochastic Galerkin FEM for log-normal coefficients in hierarchical tensor representations, preprint (2018), to appear on *Numerische Mathematik*

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The cost of controlling parabolic equations with inverse square potential or degeneracy inside

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We investigate the null controllability of the following problems:

- the heat equation with inverse square potential (a problem appearing in combustion):

$$\begin{cases} u_t - u_{xx} - \dfrac{\mu}{x^2} u = 0 & x \in (0,1),\ t \in (0,T), \\ u(0,t) = 0 & t \in (0,T), \\ u(1,t) = H(t) & t \in (0,T), \\ u(x,0) = u_0(x) & x \in (0,1), \end{cases} \tag{3}$$

- and a typical example of parabolic equation with degeneracy inside the domain:

$$\begin{cases} u_t - (|x|^\alpha u_x)_x = h(x,t)\chi_{(a,b)}(x), & x \in (-1,1) \\ u(-1,t) = 0 = u(1,t), \\ u(x,0) = u_0(x), \end{cases} \tag{4}$$

  assuming either $0 < a < b < 1$ or $-1 < a < b < 0$.

Problem (3) is well-posed (Baras-Goldstein (1983, 1984) and null controllability holds (Vancostenoble-Zuazua (2008)) if and only if $\mu \le \frac{1}{4}$. We complete ([3]) this providing precise estimates of the null controllability cost, with respect of T and μ, of the form: there exist constants independent of $T > 0$ and of $\mu \le \frac{1}{4}$ such that

$$C_{ctr-bd}(\mu, T) \le C e^{C/T} e^{-(1+\sqrt{\frac{1}{4}-\mu})^2 T/C} \Big(1 + \sqrt{\frac{1}{4} - \mu}\Big), \tag{5}$$

and

$$C_{ctr-bd}(\mu, T) \ge c e^{c/T} e^{-(1+\sqrt{\frac{1}{4}-\mu})^2 T/c} e^{-\sqrt{\frac{1}{4}-\mu}^{4/3} (\ln \sqrt{\frac{1}{4}-\mu} + \ln \frac{1}{T})/c}. \tag{6}$$

Problem (4) was studied in particular by Martin-Rosier-Rouchon (2016) (using the the flatness method), and by Fragnelli-Mugnai (2016) (using Carleman estimates). Our approach ([4]) allowed us to discover some interesting properties, both positive and negative from the point of view of null controllability. More precisely, we obtain:

- *Negative results for* $\alpha \in [1,2)$. The negative result we prove means that, when $\alpha \in [1,2)$, the degeneracy is too strong to allow the control to act on the other side of the domain with respect to the point of degeneracy. However, null controllability still holds true for those initial condition that are supported in the same region as the control.

- *Positive results for* $\alpha \in [0,1)$. Our proof of the fact that the control is sufficiently strong to cross the degeneracy point requires to use fine properties of Bessel functions, and allows us to obtain a sharp estimate of the blow-up rate of the null controllability cost as $\alpha \to 1^-$: there exists constants independent of $\alpha \in [0,1)$ and of $T > 0$ such that

$$\frac{C'}{(1-\alpha)\sqrt{T}} e^{-T/C'} \le C_{NC}(\alpha, T) \le \frac{C}{(1-\alpha)^2} e^{-T/C} e^{C/T}. \tag{7}$$

For both models, our analysis builds on the spectral analysis (the eigenelements can be expressed in terms of Bessel functions), and on the moment method, developed by Fattorini-Russel (1971), Seidman-Avdonin-Ivanov (2000), Güichal (1985), Tenenbaum-Tucsnak (2007) and Lissy (2014), precising some general existence results ([1,2]) for suitable biorthogonal families under some 'asymptotic gap conditions', and obtaining some new fine properties of the Bessel functions J_ν of large order ν.

References

[1] P. Cannarsa, P. Martinez, J. Vancostenoble, The cost of controlling weakly degenerate parabolic equations by boundary controls, *Math. Control Relat. Fields*, **Volume no.2** (2017), pp. 171-211

[2] P. Cannarsa, P. Martinez, J. Vancostenoble, Precise estimates for biorthogonal families under asymptotic gap conditions, Discrete Contin. Dyn. Syst. Ser. S, to appear.

[3] P. Martinez, J. Vancostenoble, The cost of boundary controllability for a parabolic equation with inverse square potential, Evol. Eq. Control Theory, to appear.

[4] P. Cannarsa, R. Ferretti, P. Martinez, Null controllability of parabolic equations with interior degeneracy and one-sided control, submitted, arXiv identifier 1807.00026

Joint work with: Piermarco Cannarsa (*Dipartimento di Matematica, Università di Roma "Tor Vergata"*), Roberto Ferretti (*Università Roma 3*), and Judith Vancostenoble (*Institut de Mathématiques de Toulouse, UMR CNRS 5219, Université Paul Sabatier Toulouse III*).

Long time behavior of first order mean field games on euclidean space

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The aim of this talk is to present the results obtained by the speaker about the long time behavior of solutions to deterministic mean field games systems on Euclidean space. This problem was addressed on the torus $\mathbb{T}^n$ in [P. Cardaliaguet, *Long time average of first order mean field games and weak KAM theory*, Dyn. Games Appl. 3 (2013), 473–488], where solutions are shown to converge to the solution of a certain ergodic mean field games system on $\mathbb{T}^n$. By adapting the approach in [A. Fathi, E. Maderna, *Weak KAM theorem on non compact manifolds*, NoDEA Nonlinear Differential Equations Appl. 14 (2007), 1–27], we identify structural conditions on the Lagrangian, under which the corresponding ergodic system can be solved in $\mathbb{R}^n$. Then we show that time dependent solutions converge, in some sense, to the solution of such a stationary system on all compact subsets of the whole space.

References

[1] P. Cannarsa, W. Cheng, C. Mendico, K. Wang, Long time behavior of first order Mean Field Games on Euclindean space, preprint (2018), arXiv: 1809.09057.

[2] P. Cardaliaguet, Long time average of first order mean field games and weak KAM theory, Dyn. Games Appl. **3** (2013), 473–488.

[3] A. Fathi, Weak KAM Theorem and Lagrangian Dynamics.

[4] A. Fathi, E. Maderna, Weak KAM theorem on non compact manifolds, NoDEA Nonlinear Differential Equations Appl. **14** (2007), 1–27.

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A block moments method: dealing with spectral condensation in the minimal null control time problem for parabolic systems with scalar control

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In this talk I will present recent results on the minimal null control time problem for parabolic systems with a scalar control. These results were obtained in collaboration with Assia Benabdallah and Franck Boyer.

We propose an adaptation of the classical moments method introduced by Fattorini and Russell based on the construction (and estimation) of what can be seen as a biorthogonal family by block. This generalized biorthogonal family is adapted to the specific control problem one wants to solve. In previous works concerning the minimal null control time for parabolic systems in the presence of condensation of eigenvalues the family of eigenvectors was always assumed to form a Riesz basis. While weakening this assumption we highlighted a new phenomenon: the eigenvectors can also condensate and compensate the condensation of eigenvalues thus reducing the value of the minimal null control time.

The uniform estimates we obtain allow to deal with algebraic multiplicity of eigenvalues.

References

[1] A. Benabdallah, F. Boyer and M. Morancey, A block moments method to handle spectral condensation phenomenon in parabolic control problems, preprint (2018), submitted.

Joint work with: Assia Benabdallah (*I2M, Université d'Aix-Marseille)*, Franck Boyer (*IMT, Université Toulouse III Paul Sabatier*)

Optimal Actuator Location in Semi-linear PDEs

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Finding the best actuator location to control a distributed parameter system can improve performance and significantly reduce the cost of the control. The existence of an optimal actuator location has been established for linear partial differential equations (PDEs) with various cost functions.

Nonlinearities can have a significant effect on dynamics, and such systems cannot be accurately modeled by linear models. Hence, a controller design and actuator location strategy should take nonlinear behavior into consideration. Consider a semi-linear system with state $\mathfrak{z}(t)$ on a separable reflexive Banach state space $\mathbb{Z}$:

$$\dot{\mathfrak{z}}(t) = \mathcal{A}\mathfrak{z}(t) + \mathcal{F}(\mathfrak{z}(t)) + \mathcal{B}(r)u(t), \quad \mathfrak{z}(0) = \mathfrak{z}_0 \in \mathbb{Z}, \tag{8}$$

The function $u(t)$ is the input to the system, and takes values in a Banach space $\mathbb{U}$. The control operator $\mathcal{B}$ depends on a parameter r that takes values in a set K_{ad}. The parameter r typically has interpretation as possible actuator designs. There are few theoretical studies on concurrent optimal actuator and controller design in nonlinear systems. The research also extends previous work on optimal control of nonlinear PDEs in that the linear part of the partial differential equation is not constrained to be the generator of an analytic semigroup. A general class of PDEs with weakly continuous nonlinear part is considered.

It is shown that the problem has an optimal control and actuator design. Under additional assumptions, optimality equations explicitly characterizing the optimal control and actuator are obtained. The results are applied to optimal actuator and controller design in a railway track model as well as semi-linear wave models.

References

[1] M. S. Edaletzadeh and K. A. Morris, "Stability and well-posedness of nonlinear railway track model," *IEEE Control and Systems Letters*, to appear.

[2] M. S. Edaletzadeh and K. A. Morris, "Optimal actuator design for semi-linear systems." submitted.

Joint work with: M. Sajjad Edaletzadeh (*University of Waterloo*),

Observability inequalities with compact remainder

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In this talk, based on the work [2], we show that an observability inequality with a compact remainder (equivalently, an observability inequatility on a finite co-dimensional subspace) implies an explicit spectral description of the subspace of exactly reachable states. A noteworthy consequence is that the compact remainder can be removed if (and only if) the Fattorini-Hautus test is satisfied.

This result can be applied to obtain the boundary exact controllability of many partial differential equations such as some Schrödinger equations, beam equations, Korteweg-de Vries equations, perturbed wave equations, integro-differential transport equations, etc. We will select some examples from those mentioned above and show how such observability inequalities can be produced.

References

[1] S. Dolecki and D.L. Russell, A general theory of observation and control, *SIAM J. Control Optim.*, **15** (1977), pp. 185-220

[2] M. Duprez and G. Olive, Compact perturbations of controlled systems, *Math. Control Relat. Fields*, **8** (2018), pp. 397-410

[3] V. Komornik, *Exact Controllability and Stabilization. The multiplier method*, Masson, Paris, (1994)

[4] J.-L. Lions, *Contrôlabilitée Exacte, Perturbations et Stabilisation de Systèmes Distribuées. Tome 1*, Masson, Paris, (1988)

[5] J. Peetre, Another approach to elliptic boundary problems, *Comm. Pure Appl. Math.*, **14** (1961), pp. 711-731

[6] J. Rauch and M. Taylor, Exponential decay of solutions to hyperbolic equations in bounded domains, *Indiana Univ. Math. J.*, **24** (1974), 79-86

Joint work with: Michel Duprez (*Université Pierre et Marie Curie, Paris*)

On a Model for the Growth of Tree Stems and Vines

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In this talk, we propose a model describing the growth of tree stems and vines, taking into account also the presence of external obstacles. The system evolution is described by an integral differential equation which becomes discontinuous when the stem hits the obstacle. The stem feels the obstacle reaction not just at the tip, but along the whole stem. This fact represents one of the main challenges to overcome, since it produces a cone of possible reactions which is not normal with respect to the obstacle. However, using the geometric structure of the problem and optimal control tools, we are able to prove existence and uniqueness of the solution under natural assumptions on the initial data.

References

[1] A. Bressan, M. Palladino and W. Shen, *Growth Models for Tree Stems and Vines*, Journal of Differential Equations, Volume 263, Issue 4, (2017), Pages 2280-2316.

[2] A. Bressan, M. Palladino, *Well-posedness of a Model for the Growth of Tree Stems and Vines*, Discrete Contin. Dyn. Syst. Series A, Vol. 38, Issue 4, (2018), 2047-2064.

Joint work with: Alberto Bressan, (*Penn State University*), Wen Shen, (*Penn State University*).

Tumor growth: from compressible models to free boundary problems

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Tissue growth, as in solid tumors, can be described at a number of different scales from the cell to the organ. For a large number of cells, the 'fluid mechanical' approach has been advocated recently by many authors in mathematics or biophysics. Several levels of mathematical descriptions are commonly used, including possibly elasticity, visco-elastic laws, nutrients, active movement, surrounding tissue, vasculature remodeling and several other features.

We will focuss on the links between two types of mathematical models. The 'microscopic' or 'compressible' description is at the cell population density level and a more macroscopic or 'incompressible' description is based on a free boundary problem close to the classical Hele-Shaw equation. In the stiff pressure limit, we are going to derive a weak formulation of the corresponding Hele-Shaw free boundary problem and we will make the connection with its geometric form.

Including additional features also opens other questions as circumstances in which singularities and instabilities may develop.

References

[1] B. Perthame, F. Quirós, and J.-L. Vázquez. The Hele-Shaw asymptotics for mechanical models of tumor growth. *Arch. Ration. Mech. Anal.*, 212(1):93–127, 2014.

[2] I. C. Kim, B. Perthame, and P. E. Souganidis. Free boundary problems for tumor growth: a viscosity solutions approach. *Nonlinear Anal.*, 138:207–228, 2016.

[3] P. Gwiazda, B. Perthame, and A. Świerczewska-Gwiazda. A two species hyperbolic-parabolic model of tissue growth. Preprint hal-01881950 (2018).

The turnpike property in mean field games

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The turnpike property of optimal control problems raised new interest in the last few years, in connection with stabilization effects and long time behavior of evolutionary optimality systems. In this talk, based on joint works with J.M. Lasry, P.L. Lions and P.Cardaliaguet, I will present a turnpike result which appears in mean field game theory; in the optimal control interpretation, this corresponds to the control of a Fokker-Planck equation through the action of the drift. I will especially address how the turnpike result appears by mixing global convexity, linearization techniques and decoupling methods, in which the so-called master equation (a PDE in infinite dimensions) plays a major role. All the three steps seem to be relevant in other problems where the turnpike property is expected to hold.

References

[1] Cardaliaguet P., Lasry J.-M., Lions P.-L., Porretta A., Long time average of mean field games with a nonlocal coupling, *Siam J. Control Optimization* **51** (2013), 3558-3591.

[2] Cardaliaguet P., Porretta A., Long time behavior of the master equation in mean-field game theory, preprint arXiv:1709.04215 (2017), to appear on *Anal. & PDE*

[3] Porretta A., On the turnpike property for mean field games, in *Minimax Theory and its Applications: Special Issue on Mean Field Games and related topics*, to appear.

Phase portrait control for 1D monostable and bistable reaction-diffusion equations

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I will talk about the problem of controlling some 1D parabolic semilinear equations toward steady states, either in finite time or infinite time. These equations arise in population dynamics: they are the monostable and bistable equations on $(0, L)$ for a density of individuals $0 \leq y(t, x) \leq 1$, with Dirichlet controls $u(t), v(t)$ taking their values in $[0, 1]$, namely

$$\begin{cases} y_t - y_{xx} = f(y), \\ y(t,0) = u(t),\ y(t,L) = v(t), \\ y(0) = y_0. \end{cases}$$

Here, f is a nonlinearity either of monostable (like $f(y) = y(1-y)$) or bistable (like $f(y) = y(1-y)(y-\theta)$) .

I will explain how results by Matano [1] and P.L. Lions [2] are key. They imply that that the system can never be steered to extinction (steady state 0) or invasion (steady state 1) in finite time. The system is also asymptotically controllable to 1 independently of the size L, and to 0 if the length L of the interval domain is less than some threshold value $L^\star$, which can be computed from transcendental integrals. In the bistable case, controlling to the other homogeneous steady state $0 < \theta < 1$ is much more intricate. The staircase control strategy allows us to prove that θ can be reached in finite time if and only if $L < L^\star$.

I will highlight why the phase plane of those equations is instrumental in the whole process, whence the name *phase plane control*. These results are the subject of the submitted work [3].

References

[1] H. Matano, Convergence of solutions of one-dimensional semilinear parabolic equations, *Journal of Mathematics of Kyoto University, 1978.*

[2] P.L. Lions, On the existence of positive solutions of semilinear elliptic equations, *SIAM review, 1982.*

[3] C.P., E. Trélat, E. Zuazua, Phase portrait control for 1D monostable and bistable reaction-diffusion equations, preprint arXiv (2018), submitted.

Joint work with Emmanuel Trélat, (*Sorbonne Université, Université Paris-Diderot SPC, CNRS, Laboratoire Jacques-Louis Lions, F-75005, Paris, France*) and Enrique Zuazua (*Departamento de Matemáticas, Universidad Autónoma de Madrid, 28049 Madrid, Spain; Facultad Ingeniería, Universidad de Deusto, Avda. Universidades, 24, 48007 Bilbao, Basque Country, Spain*).

Some existence results for the isoperimetric problem with double density

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In this talk we will discuss the existence of isoperimetric sets in an Euclidean space with double density. This means that two l.s.c. functions are given, namely, $f : \mathbb{R}^N \to \mathbb{R}^+$ and $g : \mathbb{R}^N \times \mathbb{S}^{N-1} \to \mathbb{R}^+$. Then, for any set $E \subseteq \mathbb{R}^N$ of locally finite (classical) perimeter, the *weighted volume* and *perimeter* are given by

$$V(E) = \int_E f(x)\,dx\,, \qquad\qquad P(E) = \int_{\partial^* E} g(x, \nu_E(x))\,d\mathcal{H}^{N-1}(x)\,,$$

respectively, where $\partial^* E$ is the reduced boundary of E and for any $x \in \partial^* E$ the outer normal vector is denoted by $\nu_E(x)$. We will describe the general question, and we will present some old and some recent results about the problem.

References

[1] G. De Philippis, G. Franzina and A. Pratelli, Existence of Isoperimetric sets with densities "converging from below" on $\mathbb{R}^N$, J. Geom. Anal. **27** (2017), no. 2, 1086–1105.

[2] F. Mogan and A. Pratelli, Existence of isoperimetric regions in $\mathbb{R}^n$ with density, Ann. Global Anal. Geom. **43** n. 4 (2013), 331–365.

[3] A. Pratelli and G. Saracco, On the isoperimetric problem with double density, *Nonlinear Analysis*, to appear (2018).

[4] A. Pratelli and G. Saracco, The ε-ε^β property for a double density, and the regularity of double-density isoperimetric sets, preprint (2018).

Joint works with: Guido De Philippis (SISSA), Giovanni Franzina (University of Firenze), Frank Morgan (Williams College), Giorgio Saracco (University of Pavia)

Optimal control of resources for species survival

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In this work, we are interested in the analysis of optimal resources configurations (typically foodstuff) necessary for a species to survive. For that purpose, we use the logistic diffusive equation

$$\begin{cases} \partial_t \theta = \mu \Delta \theta(x) + (m(x) - \theta(x))\theta(x) & t > 0,\ x \in \Omega, \\ \frac{\partial \theta}{\partial \nu} = 0 & x \in \partial\Omega, \end{cases} \tag{LDE}$$

where θ is the population densities, m stands for the resources distribution and $\mu > 0$ stands for the species velocity also called *diffusion rate*. This system models the evolution of population density involving a term standing for the heterogeneous spreading (in space) of resources. The principal issue investigated in this talk writes: *how to spread in an optimal way resources in a closed habitat?* This problem can be recast

- either as the one of minimizing the principal eigenvalue of an elliptic operator,
- or maximizing the total population size (given by $\int_\Omega \theta$)

with respect to the domain occupied by resources, under a volume constraint. By using symmetrization techniques, as well as necessary optimality conditions, we prove new qualitative results on the solutions. In particular, we investigate the optimality of particular configurations and the asymptotic behavior of optimizers as μ tends to $+\infty$.

References

[1] H. Berestycki, F. Hamel, and L. Roques. Analysis of the periodically fragmented environment model : I – species persistence. *Journal of Mathematical Biology*, 51(1):75–113, 2005.

[2] J. Lamboley, A. Laurain, G. Nadin, and Y. Privat. Properties of optimizers of the principal eigenvalue with indefinite weight and Robin conditions. *Calc. Var. Partial Differential Equations*, 55(6):55:144, 2016.

[3] Y. Lou and E. Yanagida. Minimization of the principal eigenvalue for an elliptic boundary value problem with indefinite weight, and applications to population dynamics. *Japan J. Indust. Appl. Math.*, 23(3):275–292, 2006.

[4 I. Mazari, G. Nadin, and Y. Privat. Optimal location of resources maximizing the total population size in logistic models. Preprint HAL, 2018.

Joint work with: Jimmy Lamboley, Idriss Mazari, Grégoire Nadin, (*Sorbonne univ.*), Antoine Laurain (*Sao Paulo univ.*)

Controllability and minimal time for control of the transport equation

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The talk focuses on the control of the transport equation with a localized control.

We first prove that controllability holds if a suitable Geometric Condition is satisfied. Nevertheless, we show that approximate controllability only holds, in general.

We then discuss the problem of finding the minimal time to steer the system from a configuration to another. We describe the structure of times for which it is possible, and derive a formula for the infimum time.

References

[1] M. Duprez, M. Morancey, F. Rossi, Approximate and exact controllability of the continuity equation with a localized vector field, preprint (2018), to appear on *SIAM J Control Optim.*

[2] M. Duprez, M. Morancey, F. Rossi, Minimal time problem for crowd models with a localized vector field, preprint (2018), submitted.

Joint work with: Michel Duprez (*Sorbonne Univ., France*) and Morgan Morancey (*Univ. Aix-Marseille, France*).

Gaussian Wave Packet Transform methods for the semiclassical Schrödinger equation with random inputs

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In this work, we study the semiclassical limit of the Schrödinger equation with random inputs. We show that the equation produces $O(\varepsilon)$ oscillations in the random variable space, where ε demotes the non-dimensional Planck's constant. In order to perform uncertainly quantification in this regime, standard methods require to resolve the small space and time scales, and a large number of grid points is necessary, making it very hard to produce accurate results, especially for multidimensional problems. Using the Gaussian Wave Packet transform, the original Schrödinger equation for the wave function ψ is mapped to an ODE system for the wave packet parameters coupled with a PDE for the quantity w in rescaled variables. This approach has been introduced in [1,2] for the Schrödinger equation with scalar potential, and extended in [3] to the case with vector potential. In the new variables the small scales in space and time disappear, and accurate computations are possible with a relatively small number of degrees of freedom per dimension. Here we show that the w equation does not produce ε dependent oscillations, and thus it is more amenable for numerical simulations, and for uncertainly quantification. We propose multi-level sampling strategy in implementing the Gaussian wave packet transform, where in the most costly part, i.e. simulating the w equation, it is sufficient to use ε independent samples. Assuming smoothness in the stochastic parameter space, a very effective technique for uncertainty quantification may be based on generalized polynomial chaos methods (see [4] for example). We also provide extensive numerical tests as well as meaningful numerical experiments to justify the properties of the numerical algorithm, and hopefully shed light on possible future directions.

References

[1] G. Russo and P. Smereka, The Gaussian wave packet transform: Efficient computation of the semi-classical limit of the Schrödinger equation. Part 1: a formulation and the one dimensional case, *Journal of Computational Physics*, **Volume no. 233** (2013), pp. 192-209.

[2] G. Russo and P. Smereka, The Gaussian wave packet transform: efficient computation of the semi-classical limit of the Schrödinger equation. Part 2: Multidimensional case, *J. Comput. Phys.*, **Volume no. 257** (2014), pp. 1022-1038.

[3] Z. Zhou and G. Russo, The Gaussian Wave Packets Transform for the semi-classical Schrödinger equation with vector potentials, preprint (2018), accepted on *Communications in Computational Physics.*

[4] D. Xiu and J. S. Hesthaven, High-order collocation methods for differential equa- tions with random inputs, *SIAM J. Sci. Comput.*, **Volume no. 27** (2005), pp. 1118-1139.

Joint work with: Shi Jin (*University of Wisconsin*), Liu Liu (*University of Texas at Austin*) Zhennan Zhou (*Peking University*)

Some results about stability analysis, regularizations and applications to splitting methods in optimal control

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This talk focuses on optimal control systems. We'll analyse some results concerning the stability properties under perturbations of the system in absence of the so-called Tikhonov regularization term. More precisely, we'll consider some problems (control of ODEs) exhibiting the so-called bang-bang type solutions and we'll discuss their stability properties under small perturbations. Furthermore, we'll consider an optimal control problem of linear elliptic PDEs coming from the inverse problems, and we'll discuss the use of indirect methods based on splitting techniques.

References

Joint works with: Axel Böhm, Radu Bot, Caroline Geiersbach (University of Vienna), and Vladimir Veliov (*Vienna University of Technology*).

An affine covariant composite step method for equality constrained optimization in function space

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We present a composite step method, designed for the solution of nonlinear PDE constrained optimization problems. As a distinguishing feature, the globalization mechanism depends on primal quantities only, which facilitates the incorporation of known functional analytic problem structure.

For the computation of the steps we employ a projected cg method, equipped with a block triangular constraint preconditioner. For efficiency reasons, steps are computed inexactly. The accuracy of these computations is chosen adaptively.

Finally, we study the numerical performance of our method, by applying it to a range of optimization problems subject to nonlinear partial differential equations.

Joint work with: Martin Weiser (*Zuse-Institut Berlin*), Lars Lubkoll (*Zuse-Institut Berlin*), Manuel Schaller (*Universität Bayreuth*)

Numerical Identification of Motor Units in Muscles

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High density surface Electromyography (sEMG), is a non-invasive method of measuring the activity of muscles whereby an array of electrodes is placed above the skin and a spatially and temporally resolved measurement of the electric potential on the skin is obtained. Recent advances in high-density sEMG measurement have opened the possibility of extracting information about single motor units (groups of muscle fibers controlled by the same motor neuron) from the sEMG signal.

In this talk we present a mathematical approach to identify these motor units from measurements. On base of an electrostatic forward model we discuss an optimal control approach and the corresponding function space oriented optimization methods that we use for the identification. Finally we present some numerical results.

References

[1] Sproll, Tobias, Schiela Anton and Lowery, Madeleine, Numerical Identification of Motor Units using an Optimal Control Approach, *IFAC-PapersOnLine*, **51.** (2018), pp. 174-179

Joint work with: Anton, Schiela (*University Bayreuth*), Madeleine, Lowery (*University College Dublin*)

Reduced Order Methods for Optimal Flow Control Problem with Application in Environmental Marine Sciences and Engineering

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In this talk we propose reduced order methods as a suitable tool to manage parametrized optimal control problems governed by partial differential equations. We will focus on applications in environmental marine sciences and engineering: these problems are usually studied for different parametric configurations in order to reliably describe several physical phenomena. Environmental parametrized optimal control problems require a demanding computational effort. In order to save computational time, we exploit a POD-Galerkin reduction of the optimality system in its saddle point formulation as an appropriate and rapid approach to solve this issue. Two environmental applications are presented: a pollutant control in the Gulf of Trieste, Italy and a solution tracking governed by quasi-geostrophic equations describing nonlinear North Atlantic Ocean dynamic.

References

[1] M. Strazzullo, F. Ballarin, R. Mosetti, and G. Rozza, Model Reduction for Parametrized Optimal Control Problems in Environmental Marine Sciences and Engineering, *SIAM Journal on Scientific Computing*, **40(4)** (2018), pp. B1055-B1079.

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Inverse Source Problem related to the Gravitational Waves in General Relativity

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The gravitational waves are propagations of distortion of space-time caused by, for instance, collisions of black holes. These days, its observation makes us be interested in this phenomena. The equation which the gravitational waves should satisfy is a system of quasi-linear hyperbolic type equations gained from the Einstein equation by imposing some physical assumptions. In this time, we focus on the local inverse source problem of the gravitational waves by considering the linearized equation with a source term indicating the presence of matter, and will see the result of the Hölder type stability.

References

[1] M. Bellassoued and M. Yamamoto, *Carleman Estimates and Applications to Inverse Problems for Hyperbolic Systems*, Springer Monographs in Mathematics, Springer Japan, Tokyo, (2017)

[2] P. A. M. Dirac, *General Theory of Relativity*, Princeton University Press, (1996)

A mathematical model for elastic-plastic composites and homogenization.

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In this talk, we investigate models for the elastic-plastic deformation and the homogenization theory. Our objective is to develop an efficient and effective method for determining the effective property such as SS (Strain-Stress)-curve of composite materials. Our formulation enables us to input the material law and morphography of components directly into models. Also, we develop a variational numerical approximation method for the elastic-plastic equation and the homogenization equation.

Joint work with: Kazifumi Ito (*North Carolina State University*), and Junichi Nakagawa (Mathematic Lab, Nippon Steel & Sumitomo Metal Corporation).

Positive minimal time for the control of state constrained dynamical systems

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Given a finite-dimensional linear control system satisfying the Kalman condition, or given the heat equation in a bounded domain with internal or boundary control, if there are no state or control constraints, then controllability can be achieved in arbitrarily small time.

Now, we add some state constraints and we investigate the question of knowing if one can still pass from any initial condition to any final target in arbitrarily small time. Surprisingly, this question has received little attention, even in the simplest case of finite-dimensional linear control systems. We prove that, in this very simple case, if the initial state and target state are steady states, then controllability can be achieved in time large enough, and we prove that in general there exists a positive minimal time, even for unilateral linear state constraints (like nonnegativity of the state).

In the infinite dimensional setting, we focus on the heat equation with homogeneous Dirichlet boundary conditions, which is a well known model that preserves nonnegativity. Besides, due to infinite velocity propagation, without any constraint, the heat equation is null-controllable within arbitrary small time, with controls supported in any arbitrarily open subset of the domain (or its boundary) where heat diffuses.

Considering a positive initial condition and a positive target steady-state, we prove that controllability can be achieved, keeping a nonnegative state all along the trajectory, in time large enough, and that there is a positive (i.e., nonzero) minimal time. In other words, in spite of infinite velocity propagation, realizing controllability under an unilateral nonnegativity state constraint requires a positive minimal time.

Similar results are obtained for unilateral control constraints, and also for other models, including more general parabolic models, various boundary conditions, various (internal or boundary) controls.

References

[1] J. Lohéac, E. Trélat, E. Zuazua, Minimal controllability time for finite-dimensional linear control systems under state constraints, *Automatica J. IFAC* **96** (2018), 380–392.

[2] J. Lohéac, E. Trélat, E. Zuazua, Minimal controllability time for the heat equation under unilateral state or control constraints, *Math. Models Methods Appl. Sci.* **27** (2017), no. 9, 1587–1644.

Joint work with: Jérôme Lohéac (*Univ. Nancy*), Enrique Zuazua (*Bilbao and Madrid UAM*)

Identification of an algebraic domain from a finite number of its generalized polarization tensors

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The talk is related to the inverse problem of recovering the shape of a bounded domain from its generalized polarization tensors. I will first introduce the concept of using infinitely many generalized polarization tensors as shape descriptors for general bounded domains. Then, I will present recent results on identifying an algebraic planar domain using only a finite number of its polarization tensors. The density with respect to Hausdorff distance of algebraic domains among all bounded domains invites to extend via approximation the obtained reconstruction procedure beyond its natural context. ? Based on this, I will present a new algorithm for shape recognition/classification with a few numerical illustrations.

References

1. H. Ammari, T. Boulier, J. Garnier, W. Jing, H. Kang and H. Wang, *Target identification using dictionary matching of generalized polarization tensors,* Found. Comput. Math., 14 (2014), no. 1, 27–62.

2. B. Gustafsson, C. He, P. Milanfar, and M. Putinar, *Reconstructing planar domains from their moments,* Inverse Problems, 16 (2000), 1053–1070.

Joint work with: Habib Ammari, Andries Steenkamp (*ETH, Zürich*), Mihai Putinar (*University of California at Santa Barbara*).

Isoperimetric inequalities for Steklov-Laplacian eigenvalues

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In a celebrated paper [3], Weinstock proves that the first non-zero Steklov Laplacian eigenvalue, among planar simply connected sets of given perimeter, is maximized by the disk. The resulting functional inequality was named after him and belongs nowadays to the folklore of applied mathematics. More recently Brock [2] showed that, if the perimeter constraint is replaced by a measure constraint, no topological restriction is needed, and moreover the inequality holds true in any dimension. In a recent paper [1] we show how to generalize the original Weinstock inequality in any dimension, in the class of convex sets with prescribed surface area. We use the inverse mean curvature flow to together with shape derivative arguments. The key result is the proof of a sharp isoperimetric inequality involving simultaneously the surface area, the volume and the boundary momentum of convex sets.

References

[1] D. Bucur, V. Ferone, C. Nitsch, C. Trombetti, Weinstock inequality in higher dimensions. (arxiv-preprint)

[2] F. Brock, An isoperimetric inequality for eigenvalues of the Stekloff problem. ZAMM Z. Angew. Math. Mech. 81 (2001), no. 1, pp. 6-71.

[3] R. Weinstock, Inequalities for a Classical Eigenvalue Problem, Journal of Rational Mechanics and Analysis, Vol. 3 (1954), pp. 745-753

Joint work with: Dorin Bucur (*Université Savoie Mont Blanc, Chambéry*), Vincenzo Ferone & Carlo Nitsch (Università di Napoli Federico II).

Bilinear control of parabolic evolution equations

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In the field of control theory of dynamical systems a huge amount of works is devoted to the study of models in which the control enters as an additive term (boundary or internal locally distributed controls). But these kinds of control systems are not suitable to describe processes that change their physical characteristics for the presence of the control action. This issue is quite common for the so-called *smart materials* and in many biomedical, chemical and nuclear chain reactions.

To deal with these situations, an important role in control theory is played by multiplicative (also called bilinear) controls that appear in the equations as coefficients.

A fundamental result in the bilinear control of evolution equations is the one due to Ball, Marsden and Slemrod [2] which establishes that the system

$$\begin{cases} u'(t) + Au(t) + p(t)Bu(t) = 0, \quad t \in [0,T] \\ u(0) = u_0 \end{cases} \tag{9}$$

defined on an infinite dimentional Banach space X, where $-A$ is a generator of a strongly continuous semigroup on X and $B : X \to X$ is a bounded linear operator, is not controllable.

On the other hand, when B is unbounded, the possibility of proving a positive controllability result remains open. This idea of exploiting the unboundness of the operator B was developed by Beauchard and Laurent in [4] for the Schrödinger equation and by Beauchard in [3] for the wave equation.

We prove a result of bilinear controllability for a class of abstract parabolic equations of the form

$$u'(t) + Au(t) + p(t)Bu(t) = 0, \qquad t \in [0, T]$$

where the operator $-A$ is the infinitesimal generator of an analytic semigroup of bounded linear operators on a Hilbert space and $p(\cdot)$ is the control function. I will show some applications of our result to different kinds of parabolic equations.

References

[1] Alabau-Boussouira F., Cannarsa P. and Urbani C., Bilinear control of evolution equations of parabolic type, submitted to *Journal of Evolution Equations* (2018)

[2] Ball J.M., Marsden J.E. and Slemrod M., Controllability for distributed bilinear systems, *SIAM Journal on Control and Optimization*, Vol. no. 20, n. 4, pp. 575–597 (1982), SIAM

[3] Beauchard K., Local controllability and non-controllability for a 1D wave equation with bilinear control, *Journal of Differential Equations*, Vol. no. 250, n. 4, pp. 2064–2098 (2011), Academic Press

[4] Beauchard K. and Laurent C., Local controllability of 1D linear and nonlinear Schrödinger equations with bilinear control, *J. Math. Pures Appl.*, Vol. no. 94, n. 5, pp. 520–554 (2010)

Joint work with: Fatiha Alabau-Boussouira, (*Université de Lorraine* and *Université Pierre et Marie Curie*), Piermarco Cannarsa (*Università di Roma Tor Vergata*)

Inverse problems by Carleman estimates for complex fluid dynamics

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Carleman estimates are L^2-weighted estimates for solutions to partial differential equations, where the weights include large parameters $s > 0$ and the estimates hold uniformly for all large s. It is well-known that Carleman estimates are very convenient for proving unique continuation, observability inequalities, uniqueness and stability for inverse problems of determining spatial coefficients of partial differential equations. In particular, for the applications to inverse problems based Carleman estimates, Bukhgeim and Klibanov (1981) created a methodolgy for the uniqueness and then Imanuvilov and Yamamoto (1998, 2001) established a method yielding the global stability in determining coefficients by extra boundary data of solutions. By their method, in principle, we can establish stability results for inverse problems, once we are given relevant Carleman estimates. Moreover it has been known that Carleman estimates yield also observability inequalities, which imply the exact controllability. Thus, it is essential to establish Carleman estimates.

Carleman estimates have been proved for single equations such as elliptic, parabolic, hyperbolic, Schrödinger equations and also for systems of Maxwell's equations, elasticity, viscoelasticity equations, etc., so that we have proved stability and uniqueness results for inverse problems for these partial differential equations. Recently with coauthors, I have derived Carleman estimates for linearized Navier-Stokes equations and applied to inverse problems, but still many important equations for fluid dynamics including fluid-flow structure interaction and non-Newtonian fluids are waiting the establishment of Carleman estimates and the applications to inverse problems.

The purpose of this talk is to demonstrate the wide range of applicability of the method by Carleman estimates to inverse problems for various types of fluid dynamics including

- compressible viscous fluid
- the Ericksen-Leslie model for the nematic liquid crystal flow
- Jordan-Moore-Gibson-Thompson equation

References

[1] M. Bellassoued and M. Yamamoto, *Carleman Estimates and Applications to Inverse Problems for Hyperbolic Systems*, Springer Japan, Tokyo, 2017.

Joint work with M. Bellassoued (University of Tunis El Manar, Tunisia), M. Hieber (Technical University of Darmstadt, Germany), O.Y. Imanuvilov (Colorado State University, USA), B. Kaltenbacher (University of Klagenfurt, Austria).

Internal rapid stabilization by a scalar feedback for the 1-D linear transport equation

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We use the backstepping method to study the stabilization of a 1-D linear transport equation on the interval $(0, L)$, by controlling the scalar amplitude of a piecewise regular function of the space variable in the source term:

$$\begin{cases} \alpha_t + \alpha_x + \mu\alpha = u(t)\varphi(x), \ x \in [0, L], \\ \alpha(t, 0) = \alpha(t, L), \ \forall t \geq 0, \end{cases} \tag{10}$$

with a linear feedback control in the form of

$$\langle \alpha(t), F \rangle = \sum_{n\in\mathbb{Z}} \overline{F_n}\alpha_n(t) = \int_0^L \bar{F}(s)\alpha(s)ds.$$

The backstepping method consists in mapping the system into another target system, which is known to be stable. The feedback then becomes a parameter of this backstepping transformation, and the stabilization problem consists in finding parameters for which this transformation is invertible. Initially designed in the case of boundary controlled systems (see [2,3], and [1] for a comprehensive survey), using Volterra transforms of the second kind, the method has been extended to systems with internal control, and to more general transformations of the Fredholm form.

Using this method, we prove that when the system is exactly controllable, then it can be exponentially stabilized by means of a linear feedback. For agiven decay rate, we give an explicit feedback that achieves that decay rate.

References

[1] Miroslav Krstić and Andrey Smyshlyaev, *Boundary control of PDEs. A course on backstepping designs*, Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, volume 16 of *Advances in Design and Control*,(2008)

[2] Dejan M. Bosković, Andras Balogh, and Miroslav Krstić, Backstepping in infinite dimension for a class of parabolic distributed parameter systems, *Math. Control Signals Systems*, **16(1)** (2003), pp. 44-75

[3] Andras Balogh and Miroslav Krstić, Infinite Dimensional Backstepping-Style Feedback Transformations for a Heat Equation with an Arbitrary Level of Instability, *European Journal of Control*, **8(2)** (2002), pp.165-175

[4] Jean-Michel Coron and Qi Lü, Local rapid stabilization for a Korteweg-de Vries equation with a Neumann boundary control on the right, *J. Math. Pures Appl. (9)*, **102(6)** (2014), pp.1080-1120

[5] Jean-Michel Coron and Qi Lü, Fredholm transform and local rapid stabilization for a Kuramoto-Sivashinsky equation, *J. Differential Equations*, **259(8)** (2015), pp.3683-3729

[6] Jean-Michel Coron and Hoai-Minh Nguyen, Null controllability and finite time stabilization for the heat equations with variable coefficients in space in one dimension via backstepping approach, *Arch. Ration. Mech. Anal.*, **225(3)** (2017), pp.993-1023

[7] Jean-Michel Coron, Ludovick Gagnon, and Morgan Morancey,Rapid stabilization of a linearized bilinear 1-D Schrödinger equation, *J. Math. Pures Appl. (9)*, **115** (2018), pp.24-73

Control of some models in population dynamics

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This lecture is devoted to present recent joint work in collaboration with C. Pouchol, E. Trélat and J. Zhu on the control of a bistable reaction-diffusion arising in the modelling of bilingual populations. We first analyse the possibility of controlling the system by tuning the Allee parameter to later consider in more detail the boundary control problem through a careful analysis of the phase portrait of steady states.

We shall also present some recent work in collaboration with D. Maity and M. Tucsnak (Univ. Bordeaux) on a linear system in population dynamics involving age structuring and spatial diffusion (of Lotka-McKendrick type).

References

[1] E. Trélat, J. Zhu, E. Zuazua, Allee optimal control of a system in ecology, Mathematical Models and Methods in Applied Sciences, Vol. 28, No. 9 (2018) 1665 –1697.

[2] C. Pouchol, E. Trélat, E. Zuazua, Phase portrait control for 1D monostable and bistable reaction-diffusion equations, prepint, 2018 (hal-01800382).

[3] D. Maity, M. Tucsnak, E. Zuazua, Sharp Time Null Controllability of a Population Dynamics Model with Age Structuring and Diffusion, prepint, 2018 (hal-01764865).