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UNIVERSITÀ DEGLI STUDI DI NAPOLI FEDERICO II

UNIVERSITÀ DEGLI STUDI DI FERRARA - EX LABORE FRUCTUS -

Workshop

Efficient high-order time discretization methods for PDEs

May 8-10, 2019

Villa Orlandi, Anacapri, Italy

INdAM GNCS

Gruppo Nazionale per il Calcolo Scientifico

Contents

Workshop Schedule

Wednesday May 08, 2019		
	chairman:	G. Russo
8.45-9.00		*Registration and Opening*
9.00-9.50	C.W. Shu	Strong stability of explicit Runge-Kutta time discretizations for semi-negative linear systems
9.50-10.40	D. Ketcheson	Relaxation Runge-Kutta methods: fully-discrete entropy-stability for hyperbolic PDEs
10.40-11.10		*coffee break*
11.10-12.00	Z. Jackiewicz	Strong stability preserving implicit-explicit transformed general linear methods
12.00-12.25	Z. Horvát	SSP and Positivity for Implicit Methods
12.25-12.50	B. Takács	High order numerical methods for spatial-dependent SIR models
12.50-14.15		*lunch*
	chairman:	Z. Jackiewicz
14.15-15.05	I. Higueras	Efficient Strong Stability Preserving IMEX Runge-Kutta methods
15.05-15.30	G. Albi	Linear multistep methods for optimal control problems and applications to hyperbolic relaxation systems
15.30-16.00		*coffee break*
16.00-16.50	J. Hu	A second-order asymptotic-preserving and positivity-preserving exponential Runge-Kutta method for a class of stiff kinetic equations
16.50-17.15	M. Eimer	Local time stepping scheme for district heating networks
17.15-17.40	T. Xiong	High order asymptotic preserving DG-IMEX schemes for multiscale kinetic equations

Thursday May 09, 2019		
	chairman:	C.W. Shu
9.00-9.50	J. Qiu	Semi-Lagrangian Discontinuous Galerkin Methods for Fluid and Kinetic Applications
9.50-10.40	A. Sandu	New developments in multirate integration
10.40-11.10		*coffee break*
11.10-12.00	P. Mulet	Implicit-explicit schemes for PDE with convection and degenerate diffusion
12.00-12.25	D. Conte	Exponentially fitted Peer methods for advection diffusion problems
12.25-12.50	P. Frolkovič	Semi-implicit time discretizations of advection level set equations
12.50-14.15		*lunch*
	chairman:	E. Hairer
14.15-15.05	G. Dimarco	IMEX multistep method for hyperbolic systems with relaxation
15.05-15.30	G. Puigt	Simulation of compressible flows by means of a spatially coupled implicit/explicit time integrator
15.30-16.00		*coffee break*
16.00-16.50	R.D'Ambrosio	Adapted discretization of partial differential equations generating periodic wavefronts
16.50-17.15	F.L. Romeo	A Semi-Implicit Discontinuous Galerkin Space-Time scheme on 2D Staggered meshes for Fluid Applications
17.15-17.40	G. Bertaglia	The augmented FSI system for blood flow in viscoelastic vessels solved with IMEX schemes

Friday May 10, 2019		
	chairman:	L. Pareschi
9.00-9.50	E. Hairer	Numerical long-time conservation of energy momentum and actions for nonlinear wave equations
9.50-10.40	N. Guglielmi	Numerical inverse Laplace transform for convection-diffusion equations
10.40-11.10	*coffee break*	
11.10-12.00	A. Ostermann	A low-regularity Fourier integrator for the cubic nonlinear Schrödinger equation
12.00-12.50	G. Samaey	Projective and telescopic projective integration for the nonlinear BGK and Boltzmann equations
12.50-13.00	*Closing*	

Scientific Committee

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- **Giuseppe Izzo**, Dipartimento di Matematica e Applicazioni "R.Caccioppoli", Università degli Studi di Napoli Federico II, Italy.
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Invited Talks

Adapted discretization of partial differential equations generating periodic wavefronts

Raffaele D'Ambrosio
Università degli Studi dell'Aquila, Italy

The talk focuses on the numerical solution of advection-reaction-diffusion problems by adapted finite difference schemes. The numerical scheme is developed in order to exploit the a-priori knowledge of the qualitative behaviour of the solution, gaining advantages in terms of efficiency and accuracy with respect to classical schemes already known in literature, which mostly rely on algebraic polynomials.

The adaptation is carried out by a non-polynomially fitted space-discretization and an Implicit-Explicit (IMEX) time-integration. The coefficients of the resulting numerical scheme depend on unknown parameters to be properly estimated: such an estimate is performed by an efficient offline minimization of the leading term of the local truncation error. The effectiveness of this problem-oriented approach is provided through rigorous theoretical results and selected numerical experiments.

Joint work with Beatrice Paternoster (Università degli Studi di Salerno, Italy).

IMEX multistep method for hyperbolic systems with relaxation

Giacomo Dimarco

Università degli Studi di Ferrara, Italy

In this talk, we consider the development of Implicit-Explicit (IMEX) Multistep time integrators for hyperbolic systems with relaxation. More specifically, we consider the case in which the transport and the relaxation part of such systems have different time and space scales. The consequence is that the nature of the asymptotic limit can differ, passing from an hyperbolic to a parabolic character. From the computational point of view, this causes many drawbacks that standard time integrators, even implicit ones, are not able to handle: loss of efficiency and loss of capability in describing the limit regime. In this work, we construct highly stable numerical methods which describe all the different regimes with high accuracy in time and space that are able to capture the correct asymptotic limit. Several numerical examples confirm the consistency and linear stability analysis and show that the proposed methods outperform existing ones.

Numerical inverse Laplace transform for convection-diffusion equations

Nicola Guglielmi

Gran Sasso Science Institute, L'Aquila, Italy

In this talk a novel contour integral method is proposed for linear convection-diffusion equations. The method is based on the inversion of the Laplace transform and makes use of a contour given by an elliptic arc joined symmetrically to two half-lines. The trapezoidal rule is the chosen integration method for the numerical inversion of the Laplace transform, due to its well-known fast convergence properties when applied to analytic functions. Error estimates are provided as well as careful indications about the choice of several involved parameters. The method selects the elliptic arc in the integration contour by an algorithmic strategy based on the computation of pseudospectral level sets of the discretized differential operator. In this sense the method is general and can be applied to any linear convection-diffusion equation without knowing any a priori information about its pseudospectral geometry. Numerical experiments performed on the Black-Scholes (1D) and Heston (2D) equations show that the method is competitive with other contour integral methods available in the literature.

Joint work with Maria Lopez Fernandez (Università di Roma La Sapienza, Italy) *and* Giancarlo Nino (Université de Genève, Switzerland).

Numerical long-time conservation of energy, momentum and actions for nonlinear wave equations

Ernst Hairer

Université de Genève, Switzerland

This talk considers nonlinearly perturbed wave equations, pseudo-spectral semi-discretizations and full discretizations using trigonometric time integrators. The long-time near-conservation of energy, momentum, and harmonic actions is studied. Rigorous statements are shown under suitable numerical non-resonance conditions and under a CFL condition. The time step is not assumed to be small compared to the inverse of the largest frequency in the space-discretized system, so that classical backward error analysis cannot be applied. The proofs of the statements on the long-time conservation properties are based on the technique of modulated Fourier expansions.

Joint work with Christian Lubich (University of Tübingen, Germany) *and* David Cohen (Umeå University, Sweden).

Related publications can be downloaded from http://www.unige.ch/~hairer/preprints.html

Efficient Strong Stability Preserving IMEX Runge-Kutta methods

Inmaculada Higueras

Universidad Pública de Navarra, Pamplona, Spain

During the last decades, the study of Strong Stability Preserving (SSP) properties (e.g., monotonicity, contractivity, positivity, discrete maximum principles, etc.) for Runge-Kutta methods has been an active research topic. Different results are available in the literature and several numerical experiments have been performed with this kind of methods. However, in terms of efficiency, besides the SSP properties of a scheme some other topics turn out to be relevant. These include, e.g. type of problems, sharpness of the theoretical step size restrictions, implementation issues, etc. In this talk we will discuss these issues for SSP Implicit-Explicit Runge-Kutta methods.

A second-order asymptotic-preserving and positivity-preserving exponential Runge-Kutta method for a class of stiff kinetic equations

Jingwei Hu

Purdue University, West Lafayette, IN, USA

We introduce a second-order time discretization method for stiff kinetic equations. The method is asymptotic-preserving (AP) - can capture the Euler limit without numerically resolving the small Knudsen number; and positivity-preserving - can preserve the non-negativity of the solution which is a probability density function for arbitrary Knudsen numbers. The method is based on a new formulation of the exponential Runge-Kutta method and can be applied to a large class of stiff kinetic equations including the BGK equation (relaxation type), the Fokker-Planck equation (diffusion type), and even the full Boltzmann equation (nonlinear integral type). Furthermore, we show that when coupled with suitable spatial discretizations the fully discrete scheme satisfies an entropy-decay property. Various numerical results are provided to demonstrate the theoretical properties of the method.

Joint work with Ruiwen Shu (University of Maryland).

Strong stability preserving implicit-explicit transformed general linear methods

Zdzislaw Jackiewicz

Arizona State University, Tempe, AZ, USA, and
AGH University of Science and Technology, Krakow, Poland

For many systems of differential equations modeling problems in science and engineering, there are often natural splittings of the right hand side into two parts, one of which is non-stiff or mildly stiff, and the other part is stiff. Such systems can be efficiently treated by a class of implicit-explicit (IMEX) general linear methods (GLMs), where the stiff part is integrated by implicit formula, and the non-stiff part is integrated by an explicit formula. We will construct methods where the explicit part has strong stability preserving (SSP) property, and the implicit part of the method has inherent Runge-Kutta stability (IRKS) property, and it is A-, or L-stable. We will also investigate stability of these methods when the implicit and explicit parts interact with each other. To be more precise, we will monitor the size of the region of absolute stability of the IMEX scheme, assuming that the implicit part of the method is $A(\alpha)$-stable for $\alpha \in [0, \pi/2]$. Finally we furnish examples of SSP IMEX GLMs up to the order $p = 4$ and stage order $q = p$ with optimal SSP coefficients.

Joint work with Giuseppe Izzo (Università di Napoli Federico II, Italy).

Relaxation Runge-Kutta methods: fully-discrete entropy-stability for hyperbolic PDEs

David Ketcheson

King Abdullah University of Science and Technology, Saudi Arabia

Recent advances have enabled the development of efficient high-order entropy-stable discretizations for the Euler and Navier-Stokes equations. However, the strict entropy-stability property is destroyed by standard explicit time discretizations. I will present a class of Runge-Kutta-like methods, related to projection methods, that guarantee conservation or stability with respect to any inner-product norm, and thus provide fully-discrete entropy stability for symmetric hyperbolic systems at the same cost as standard explicit Runge-Kutta time stepping. Because of the methods special form, they retain many desirable properties (including order of accuracy, approximate linear stability, and strong stability preservation) of the original Runge-Kutta method. I will show several numerical examples, including an extension to preservation of stability for arbitrary convex entropies such as the standard entropy for the Euler equations.

Joint work with H. Ranocha (TU Braunschweig, Germany), M. Alsayyari, M. Parsani, *and* L. Dalcin (KAUST, Saudi Arabia).

Implicit-explicit schemes for PDE with convection and degenerate diffusion

Pep Mulet

Universitat de València, Spain

When using the method of lines for PDE with convection and (possibly strong) degenerate diffusion, Implicit-Explicit (IMEX) Runge-Kutta (RK) methods, that combine an explicit RK scheme for the time integration of the convective part with a diagonally implicit one for the diffusive part, are suitable for the much more favourable stability restrictions with respect to explicit integrators and for not having to deal, as fully implicit solvers do, with the fairly sophisticated discretization of the convective terms when they are dominant.

In [Burger, Mulet, Villada, SISC, 2013] a scheme of this type is proposed, for which the nonlinear and nonsmooth systems of algebraic equations arising in the implicit treatment of the degenerate diffusive part are solved by smoothing of the diffusion coefficients combined with a damped Newton-Raphson method with a line search strategy for globalizing convergence. To overcome the CPU and implementations costs of these schemes while keeping the advantageous stability properties of IMEX-RK methods, a second variant of these methods is proposed in [Boscarino et al., SISC, 2015], in which the diffusion terms are discretized in a way that more carefully distinguishes between stiff and nonstiff dependence, such that in each Runge-Kutta stage only a linear system needs to be solved, still maintaining high order accuracy in time. These schemes may be advantageous in some cases, but are not advisable in those cases where special structure of the diffusive terms would be lost. This is the case of some nonlinear convection-diffusion equations with nonlocal flux and possibly degenerate diffusion that arise in many scientific contexts [Carrillo, Chertock, Huang, CCP, 2015], [Burger, Inzunza, Mulet, NMPDE, 2019].

In this talk a survey of these techniques will be given, some recent successful applications of them will be reported and some future applications, as multispecies nonlinear nonlocal equations with cross-diffusion or Navier-Stokes-Cahn-Hilliard equations, will be presented.

A low-regularity Fourier integrator for the cubic nonlinear Schrödinger equation

Alexander Ostermann
University of Innsbruck, Austria

A new filtered low-regularity Fourier integrator for the cubic nonlinear Schrödinger equation is presented. This scheme has better convergence rates at low regularity than any known scheme in the literature so far. To prove this superior error behavior, we combine the better local error properties of the new scheme with a stability analysis based on general discrete Strichartz-type estimates. The latter allow us to handle a much rougher class of solutions as the error analysis can be carried out directly at the level of L^2. We are able to establish a global error estimate in L^2 for H^1 solutions, which is roughly of order $\tau^{\frac{1}{2}+\frac{5-d}{12}}$ in dimension $d \leq 3$ with τ denoting the time step size.
For details, see https://arxiv.org/pdf/1902.06779.pdf .

Joint work with Frédéric Rousset (Université Paris-Sud, France) *and* Katharina Schratz (KIT, Karlsruhe, Germany).

Semi-Lagrangian Discontinuous Galerkin Methods for Fluid and Kinetic Applications

Jingmei Qiu
University of Delaware, Newark, DE, USA

We propose Semi-Lagrangian discontinuous Galerkin (SLDG) schemes for convection-diffusion and convection-relaxation problems with fluid and kinetic applications. The classical grid-based Eulerian methods, e.g. finite difference (FD) methods, finite volume (FV) methods and DG methods, can achieve arbitrary spatial and temporal order of accuracy, yet they suffer quite stringent time stepping size restriction with explicit time-stepping methods. In order to be free of the time step constraint, we propose to use the grid-based SL approach, which propagates information along characteristics, allowing very large CFL numbers and leading to computational efficiency. Due to its efficiency property, SL schemes are widely used in incompressible flows, plasma physics, and global multi-tracer transport in atmospheric modeling.

For fluid problems, such as linear convection-diffusion, we propose to apply the SLDG [Guo, Nair and Qiu, MWR, 2014] method to the convection term, together with the LDG discretization of the diffusion term coupled with diagonally implicit RK (DIRK) time discretization along characteristics. For the nonlinear incompressible Navier-Stokes equation, backward characteristics tracing with high order accuracy could be challenging. We propose to apply the RK exponential integrator [Celledoni and Comet, JSC, 2009], to frozen the nonlinear advection coefficients and to couple with implicit treatment of linear diffusion terms. Our proposed schemes are mass conservative, truly multi-dimensional without dimensional splitting errors, genuinely high order accurate in both space and time, and highly efficient by allowing extra large time stepping size.

For kinetic problems, such as the BGK equation, we propose to treat the convection term by the SLDG method, while the relaxation term is evolved with DIRK methods along characteristics. Our schemes enjoy mass conservation, high order space-time accuracy and are free of the CFL constraint. Moreover, our proposed schemes possess asymptotic-preserving property which preserves the asymptotic Euler limit as the Knudsens number going to zero. The performance is showcased by several benchmark problems.

Projective and telescopic projective integration for the nonlinear BGK and Boltzmann equations

Giovanni Samaey

KU Leuven (University of Leuven), Belgium

We present high-order, fully explicit projective integration schemes for nonlinear collisional kinetic equations such as the BGK and Boltzmann equation. The methods first take a few small (inner) steps with a simple, explicit method (such as direct forward Euler) to damp out the stiff components of the solution. Then, the time derivative is estimated and used in an (outer) Runge-Kutta method of arbitrary order. The procedure can be recursively repeated on a hierarchy of projective levels to construct telescopic projective integration methods. Based on the spectrum of the linearized collision operator, we deduce that the computational cost of the method is essentially independent of the stiffness of the problem: with an appropriate choice of inner step size, the time step restriction on the outer time step, as well as the number of inner time steps, is independent of the stiffness of the (collisional) source term. In some cases, the number of levels in the telescopic hierarchy depends logarithmically on the stiffness. We illustrate the method with numerical results in one and two spatial dimensions.

Joint work with Ward Melis (KU Leuven, Belgium)*, and* Thomas Rey (LPP - Laboratoire Paul Painlevé, RAPSODI)*.*

New developments in multirate integration

Adrian Sandu
Virginia Tech, Blacksburg, VA, USA

Multirate methods seek to solve efficiently dynamical systems characterized by multiple time scales. The solution strategy is to use small step sizes to discretize the fast components, and large step sizes to discretize the slow components. This talk will discuss the new family of multirate infinitesimal GARK schemes, where the fast component is advanced by solving an appropriate ODE, thereby allowing extreme flexibility in practical implementations. The talk will also discuss new multirate General Linear Methods.

Strong stability of explicit Runge-Kutta time discretizations for semi-negative linear systems

Chi-Wang Shu
Brown University, Providence, RI, USA

We study the strong stability property of explicit Runge-Kutta time discretizations of linear semi-discrete schemes which have semi-negative stability. It is well known that the three stage, third order Runge-Kutta method is strongly stable for semi-negative linear systems, however we show by a simple counter example that the classical four stage, fourth order Runge-Kutta method is not strongly stable, and we also show the (somewhat surprising) result that after two time steps the fourth order Runge-Kutta method is strongly stable. Furthermore, we present a general framework on analyzing the strong stability of explicit Runge-Kutta time discretizations for semi-negative autonomous linear systems. The analysis is based on the energy method and can be performed with the aid of a computer. Strong stability of various Runge-Kutta methods, including a sixteen-stage embedded pair of order nine and eight, has been examined under this framework. Based on numerous numerical observations, we further characterize the features of strongly stable schemes. A both necessary and sufficient condition is given for the strong stability of Runge-Kutta methods of odd linear order.

Joint work with Zheng Sun (Brown University, Providence, RI, USA).

Contributed Talks

Linear multistep methods for optimal control problems and applications to hyperbolic relaxation systems

Giacomo Albi
Università degli Studi di Verona, Italy

We are interested in high-order linear multistep schemes for time discretization of adjoint equations arising within optimal control problems. First we consider optimal control problems for ordinary differential equations and show loss of accuracy for AdamsMoulton and AdamsBashforth methods, whereas BDF methods preserve high-order accuracy. Subsequently we extend these results to semi-Lagrangian discretizations of hyperbolic relaxation systems. Computational results illustrate theoretical findings.

Joint work with Lorenzo Pareschi (Università degli Studi di Ferrara, Italy) *and* Michael Herty (RTWH Aachen, Germany).

The augmented FSI system for blood flow in viscoelastic vessels solved with IMEX schemes

Giulia Bertaglia

Università degli Studi di Ferrara, Italy

It is nowadays well-established that mathematical models are a powerful resource in the field of hemodynamics, being frequently adopted for various medical applications to obtain data that otherwise would require invasive measurements. The theory behind blood flow is closely related to the study of incompressible flow trough compliant thin-walled tubes. The correct numerical treatment of the fluid-structure-interaction (FSI) system of equations becomes even more challenging if we consider that veins are collapsible under certain circumstances. Recent works also showed the benefits of modeling the rheological behavior of the vessel wall using a viscoelastic law, considering in this manner that it manifests not only an instantaneous elastic strain but also a viscous damping effect, applied to pulse pressure waves, coupled to the definition of a relaxation time of the material. In this context, the purpose of this work is to propose an easily extensible 1D mathematical model able to correctly capture the FSI mechanisms inside both arterial and venous systems. The model is solved with an efficient and robust second-order numerical scheme with a time integration based on an Implicit-Explicit (IMEX) Runge-Kutta (RK) approach, proposed by Pareschi and Russo for applications to hyperbolic systems with stiff relaxation terms. The validation of the proposed model is done in different manners. Results obtained in Riemann Problems (RP), adopting a simple elastic tube law for the characterization of the vessel wall, are compared to available exact solutions. To validate also the contribute given by the viscoelasticity, a test problem for a modified non-linear system of equations that is a perturbation of the original system via a source term vector has been structured, applying it to a generic artery and to a generic vein. Specific tests have been chosen then to verify the well-balancing and the expected order of accuracy of the scheme. Finally, results obtained with different single-vessel configurations are compared to results obtained with other blood flow models and even with real medical measurements.

Joint work with Valerio Caleffi *and* Alessandro Valiani (Department of Engineering, University of Ferrara), Adrián Navas *and* Javier Murillo (Department of Science and Technology of Materials and Fluids (EINA), University of Zaragoza).

Exponentially fitted Peer methods for advection diffusion problems

Dajana Conte
Università degli Studi di Salerno, Italy

We consider advection-diffusion problems whose solution exhibits an oscillatory behaviour, such as the Boussinesq equation [1]. The semi-discretization in space of such equation gives rise to a system of ordinary differential equations, whose dimension depends on the number of spatial points. We present a general class of exponentially fitted two step peer methods for the numerical integration of ordinary differential equations having oscillatory solutions [2, 3]. Such methods are able to exploit a-priori known information about the qualitative behaviour of the solution in order to efficiently furnish an accurate solution. Moreover peer methods are very suitable for a parallel implementation, which may be necessary when the number of spatial points increases. The effectiveness of this problem-oriented approach is shown through numerical tests on well-known problems.

Joint work with Beatrice Paternoster (Università degli Studi di Salerno, Italy) *and* Leila Moradi (University of Hormozgan, Iran) .

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Local time stepping scheme for district heating networks

Matthias Eimer
Fraunhofer ITWM, Kaiserslautern, Germany

As an effective and sustainable alternative to conventional heating systems, district heating has a huge potential, especially in urban areas. In order to optimally control the use of resources, a fast and accurate forward simulation is important. In this talk we want to present a new solver for simulations of district heating networks. The numerical method applies the local time stepping that was introduced in [1] and used for blood flow models in [2] to networks of linear advection equations. Numerical diffusion as well as the computational effort on each edge is reduced significantly. In combination with a high order coupling approach an accurate and very efficient scheme is developed. In several numerical test cases we illustrate the efficiency of the method for simulations of district heating networks.

Joint work with Raul Borsche (TU Kaiserslautern, Germany) .

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Semi-implicit time discretizations of advection level set equations

Peter Frolkovič

Slovak University of Technology, Slovakia

Semi-implicit schemes for numerical solution of level set advection equations will be presented. An accuracy and a stability study is provided for a linear advection equation with a variable velocity on Cartesian grids using partial Lax-Wendroff procedure and numerical von Neumann stability analysis [1]. The obtained parametric class of semi-implicit schemes is at least 2nd order accurate in space and time and it has significantly better stability properties of von Neumann type (including unconditional stability) than analogous fully implicit or fully explicit schemes. The numerical schemes are derived and used also for polyhedral grids and for nonlinear level set advection equation [2, 3]. A main area of applications of such schemes are problems on implicitly given (dynamic) computational domains and for relaxation methods to find (nearly) stationary solutions of advection dominated problems.

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SSP and Positivity for Implicit Methods

Zoltán Horváth
Széchenyi István University, Hungary

The state-of-the-art theory for strong stability preservation (SSP) and forward invariance of sets (positivity) relies on the same property of the Forward Euler (FE) method with a positive step size. However, this theory does not apply in many cases of practical importance (e.g. FEM without lumping, spectral methods). In this talk we shall present results for diagonally implicit methods (e.g. diagonally implicit Runge-Kutta methods) that use assumptions only on the property of the Backward Euler (BE) method, instead of the FE condition. We quantify the SSP/positivity step sizes in terms of a measure of the BE-step on the given problem and a new parameter of the scheme. In addition, the new arguments propose an efficient implementation procedure as well. To illustrate the new theoretical findings, we shall present results of computational experiments for CFD problems arising in automotive industry.

Simulation of compressible flows by means of a spatially coupled implicit/explicit time integrator

Guillaume Puigt

ONERA - DMPE, France

Reynolds Averaged Navier-Stokes (RANS) equations and Large Eddy Simulation (LES) are coupled spatially by means of the DES technique. The goal is to use the RANS approach to capture the boundary layer, where most of grid points are located in a resolved LES, and to capture the turbulent unsteadiness by the LES far from the wall. Today, such a procedure is limited by the time integration procedure. Indeed, a standard framework follows an explicit time integration which induces a strong constraint on the maximum stable time step of the unsteady simulation. In order to overcome this limitation, we have worked on a coupling procedure between Heun's scheme for explicit time integration and the second-order Crank-Nicolson scheme for implicit time integration. The main aspect of the proposed work is to couple both schemes spatially and to authorize a smooth transition between the schemes on the mesh. In this context, we will present theoretical results on the proposed scheme called AION scheme (stability, Von Neumann analysis, negative group velocity) and several test cases of increasing complexity will be introduced and commented.
The present work is also the core of a paper submitted for publication in the Journal of Computational Physics.

Joint work with L. Muscat (PhD Student, CERFACS and ArianeGroup), G. Puigt (PhD advisor, ONERA), M. Montagnac (PhD co-advisor, CERFACS) *and* P. Brenner (CFD expert, ArianeGroup) .

A Semi-Implicit Discontinuous Galerkin Space-Time scheme on 2D Staggered meshes for Fluid Applications

Francesco Lohengrin Romeo
Università degli Studi di Trento, Italy

In this talk we present a high order accurate semi-implicit Discontinuous Galerkin space-time method on staggered unstructured grids, for the simulation of viscous incompressible flows on two-dimensional domains. The designed scheme is of the Arbitrary-Lagrangian-Eulerian type, which is suitable to work on fixed as well on moving meshes that are allowed to move with an almost arbitrary mesh velocity. In our space-time formulation, by expressing the numerical solution in terms of piecewise space-time polynomials, an arbitrary high order of accuracy in time is achieved through a simple and efficient method of Picard iterations. For the dual mesh, the basis functions consist in the union of continuous piecewise polynomials on the two subtriangles within the quadrilaterals: this allows the construction of a quadrature-free scheme, resulting in a very efficient algorithm, not only from a computer memory point of view, but also concerning the required CPU time. Several numerical examples confirm that the proposed method outperform existing ones.

High order numerical methods for spatial-dependent SIR models

Bálint Takács

Eötvös Loránd University (ELTE), Hungary

The SIR model, first introduced by Kermack and McKendrick in 1927 can be used to describe any process in which some property is transferred among a group of individuals. During the process, we distinguish three classes. The first one contains the individuals which have not acquired the property yet. The second class includes the individuals that possess the property and have the ability to pass it on to others. The last class contains the individuals which used to have the property but cannot transmit it any more. Such latter processes include epidemics (the process for which the first model was initially introduced) or other phenomena, for example, a fire in a forest. The original system of ordinary differential equations is extended introducing a spatial dependence, resulting in a system of partial integro-differential equations. In this talk, we show several numerical models arising from this continuous system. First, we use different techniques to discretize the problem in space and then solve the resulting system of ordinary differential equations using high order time integration methods. We prove that the explicit Euler method preserves the qualitative properties of the original biological model under a step size restriction. Then, we show that high order strong stability preserving (SSP) methods can be used to numerically solve the problem with augmented step size upper bounds. The efficiency of the theoretical results is demonstrated in numerical experiments.

High order asymptotic preserving DG-IMEX schemes for multiscale kinetic equations

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Kinetic equations which arise from dilute gas dynamics or plasma physics has a great challenge for numerical simulations. It is mainly due to its multiscale structure and high dimensionality. Asymptotic preserving (AP) schemes have been proposed for the multiscale structure. In our work, we use the discontinuous Galerkin method in space coupled with a globally stiffly accurate IMEX scheme in time, to achieve good AP limits. Numerical experiments demonstrate the efficiency and effectiveness of our proposed approach.

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