A Scalable Numerical Algorithm for solving Tikhonov Regularization Problems in a large scale application

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We are concerned with large scale Tikhonov Regularization (TR) problems. TR is the most commonly used method of regularization for inverse and ill-posed problems. There are numerous examples of ill posed problems in computational mathematics and applications. The issue we face here is to solve large scale inverse ill posed problems efficiently. Efficiency is achieved by virtue of designing computational models specifically to take full advantage of massively parallel computers and General Purpose Graphics Processing Units (GPGPUs).

1. The Tikhonov-Regularized (TR) formulation

The TR problem can be described as following:

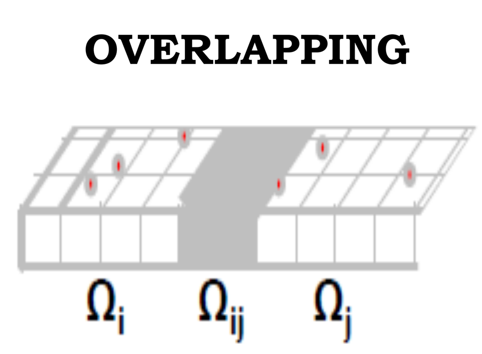
$$u(\lambda) = argmin_u J(u) = arming_u \left\{ \|\mathbf{Hu} - \mathbf{v}\|_R^2 + \lambda \|\mathbf{u} - \mathbf{u}^{\mathcal{M}}\|_B^2 \right\}$$

where $\|\cdot\|_R$ and $\|\cdot\|_B$ denote the weighted norms with respect to the error covariance matrices $\mathbf{B}$ and $\mathbf{R}$ and λ is the regularization parameter

2. The decomposed TR formulation

Let $\Omega \in R^3$ be the domain decomposed into a sequence of overlapping sub-domains $\Omega_i \in R^3$, such that:

$$\Omega = \bigcup_{i=1}^{p} \Omega_i \quad \Omega \subset R^3$$



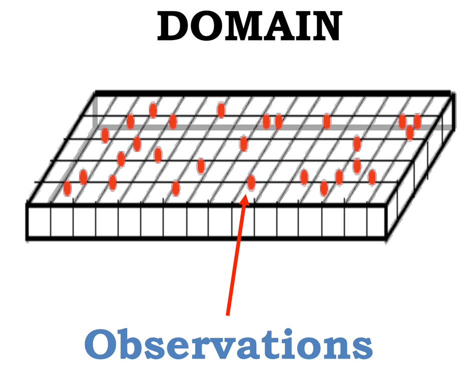
The decomposed TR formulation:

$$u_i^{DA} = argmin_{ui}\{J/\Omega_i + O/\Omega_{ij}\}$$

3. Performance Analysis – ScaleUp factor

If $p \in N$, and $p > 1$, the algorithm associated to the decomposition given is:

$$\mathcal{A}(\Omega, p) := \underbrace{\mathcal{A}(\Omega_1), \mathcal{A}(\Omega_2), \ldots, \mathcal{A}(\Omega_p)}_{\text{p times}}$$



Let $p1, p2 \in N$ and $p_1 < p_2$. Let $T(A(\omega, p_i)), i = 1, 2$ denote the time complexity of $A(\omega_i, p_i), i = 1, 2. \forall i \neq j$ we define the (relative) scale-up factor of $A(\omega, p_2)$, in going from p1 to p2, the following ratio:

$$S_{p2,p1}(N) = \frac{T(\mathcal{A}(\Omega, p1))}{(p2/p1)T(\mathcal{A}(\Omega, p2))}$$

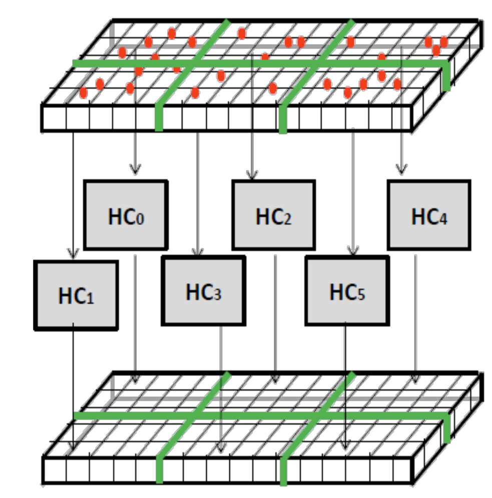
4. Case Study: Data Assimilation problem

Let $t \in [0, T]$ denote the time variable. Let $u^{true}(t, x)$ be the evolution state of a predictive system governed by the mathematical model M with $u^{true}(t_0, x), t_0 = 0$ as initial condition. Here we consider a 3D shallow water model. Let $v(t, x) = H(u_{true}(t, x))$ denote the observations mapping, where H is a given nonlinear operator which includes transformations and grid interpolations

5. Results

We consider two hybrid architectures: HA1 is a 288 CPU-multicores, HA2 is a GPU+CPU architecture

n	nproc	$T^{nproc}(N)$	$measuredS_{nproc,8}$	$S_{nproc,8}$
$O(10^6)$	8	2.0545e+02	1.0	1
	16	6.3316e+01	3.25	4
	32	2.0005e+01	10.27	16
	64	8.7835e+00	23.39	64
n	**nproc**	**$T^{nproc}(N)$**	**$measuredS_{nproc,8}$**	**$S_{nproc,8}$**
$O(10^7)$	8	-	-	-
	16	3.9091e+03	1.0	1
	32	9.9952e+02	3.91	4
	64	2.7584e+02	14.17	16



Results obtained on HA1 for a problem size $O(10^6)$ and $O(10^7)$ by using # thread-blocks = 2p

N	p	nproc	$T_{flop}^{GPU}(N)$
$O(10^7)$	1	2	0.144
	2	4	0.044
	4	8	0.025
	8	16	0.024

Values of execution time of algorithm running on GPU for a problem size $O(10^7)$

N	p	$T_{flop}^{GPU}(N)$	$measuredS_{nproc,2}$	$S_{nproc,2}$
$O(10^7)$	1	0.127	-	-
	2	0.027	4.7	4
	4	0.008	15.9	8
	8	0.007	18.1	16

Results on HA2: Values of T_{flop}^{GPU} and measured scale-up factor compared with theoretical once

6. Discussion

We now discuss scalability results shown in the tables. To this end, we introduce

$$s_{nproc}^{loc} = \frac{T_{flop}(N/p)}{T_{nproc}(N/p)}$$

which denotes the speed up of the (local) algorithm $\mathcal{A}(D_N(\Omega_i), N/p)$ for solving the local problem on subdomain $D_N(\Omega_i)$. Let us express the measured scale up factor in terms of s_{nproc}^{loc} . We have:

$$S_{1,nproc}^{measured} = \frac{T_{flop}(N)}{\frac{pT_{flop}(N/p)}{s_{nproc}^{loc}} + pT_{oh}(N/p)}$$

it comes out that

$$S_{1,nproc}^{measured} = \alpha S_{1,nproc} \qquad \alpha := \frac{s_{loc}^{nproc}}{1 + \frac{s_{nproc}^{loc} T_{oh}(N/p)}{T_{flop}(N/p)}}$$

Finally, it is worth noting that in our experiments, on HA1, local DA problems are sequentially solved, then $s_{nproc}^{loc} = 1$ and $\alpha < 1$, while on HA2, local DA problems have been concurrently solved on the GPU device, so $s_{nproc}^{loc} > 1$ and $\alpha > 1$,thus the above analysis validates the experimental results

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